Tanner

THE MARROW OF THE MATHEMATICKS,

627

Contracted into a ſmall Compaſs, and made plain and eaſie to the underſtanding of any ordinary Capacity.

CONTAINING

The Doctrines of *Arethmetick*, *Geometry*, *Aſtronomy*, *Gauging*, the Uſe of the *Sector*, *Surveying*, *Dyalling*, and the Art of *Navigation*, *&c.* Illuſtrated with ſeveral Cuts for the better Explanation of the whole Matter.

After a New, Compendious, Eaſie Method,

By *W. Pickering*, Merchant-Adventurer.

Licenſed *Octob. 9th* 1685. *ROB. MIDGLEY*

Ex Angulis Latera, vel ex Lateribus Angulos & mixtim, in Triangulis tam planis, quam Sphæricis aſſequi; Summa eſt gloria Mathematici. *Sic enim & Cœlum & Terras, & Maria fælici & admirando Calculo menſurat.* Fran. Vieta.

LONDON,

Printed by *John Richardſon* for the Author, and are to be Sold by *Tho. Paſſinger*, at the *Three Bibles* on *London-Bridge*, and *Tho. Sawbridge* at the Three *Flower-de-Luces* in *Little-Britain*. 1686.

Thom. Tanner.

TO THE INGENIOUS READER.

THAT which deters many from attempting, and the moſt from proceeding in theſe Studies, is, That they are wrapt up in ſuch a Labyrinth of hard Words, and difficult Terms of Art; that a man may be ſaid firſt to learn a Language before he can come to the things themſelves: And when he hath cracked the ſhell, and comes to the Kernel, the ſubſtance of the matter doth lye in ſuch a Chaos of Confuſion, I mean in ſuch a ſcattered immethodical, and unintelligible way of handling the ſeveral parts apart, that how theſe do lye in the whole Body, or how one piece depends upon another; for a young beginner is extream difficult to underſtand, if not paſt finding out: Indeed

ſeveral men have writ excellently well of ſeveral things, and ſome have writ of all the ſeveral parts of the Mathematicks, *Topically, or independently under ſeveral diſtinct Heads; but I have not yet met with any perfect Analyſis of the whole, or ſuch a Co-hærent continued Method in theſe Mathematical Arts as could be wiſhed; In this ſmall piece (therefore) I have indeavoured to remedy theſe inconveniences, and ſupply what is defective. Firſt, For the Terms of Art, and thoſe affected miſterious waies of expreſſion, I have either avoided them totally, or explained them ſufficiently, though whilſt I indeavour to ſpeak more plainly, perhaps by ſome I may be thought to ſpeak leſs Learnedly; but if hereby I attain the end I aim at, I matter not; which is to be underſtood by any that underſtand Common Senſe, and Plain Engliſh.*

In the ſecond place, I have endeavoured to bring the Mathematicks into one entire Method, whereby you may ſee how neceſſarily one part is Co-hærent and dependent upon another; and ſuch as deſire to ſtudy them, proceeding Methodically from

one

one thing to another, they may be greater Proficients in a few dayes, than otherwise in many Moneths, nay perhaps then, in all their Lives, without such a Method. Lastly, In all this, I have used so much brevity, and withal, such perspicuity, that (according to my own weak Judgment and Understanding) there is nothing inserted that could be wanting, nor any thing wanting that need be inserted: In a word, in the reading and studying many Authors, what I found excellent in any of them, I collected, and in most abbreviated, indeavouring in fewer words to make it more plain and easie to understand: And afterwards I have reduced this heap of confused Collections or Miscellaneas into one perfect and entire Method, in every thing indeavouring to make the Body of the Discourse correspondent to the Title-page: Hoping that this will answer both my own and the Ingenious Readers just expectation, and shew both a nearer and clearer way to the understanding the Mathematicks: *That so all may be allured by the plainness and easiness of these Arts, and Persons of ingenuity inamoured on them*

for their uſefulneſs and delight; and having ſo readily paſſed the Porch, and got over the Threſhold, they may have both time and opportunity to build rare Superſtructures upon theſe plain Foundations, and may improve theſe Arts even to miraculous Effects for the finding out many rare Inventions yet unknown to the World; whereas many (otherwiſe excellent wits) thorough their abſtruſeneſs have ſpent their whole time before they could attain to the underſtanding theſe very Rudiments. And hereupon I thought this not an unneceſſary, nor an unuſeful piece of work; and I doubt not but it will be acceptable to ſome, tho' many I know will be ready to cenſure both the Book and the Author; As for my ſelf, I have been ſo accuſtomed to be miſconſtrued in all my Actions, that the wonder would be, not to be ſo in this; and indeed I do ſo little value it, that I can ſmile in diſguiſe, to ſee the moſt ſo much miſtaken, indeavouring in the mean time (as near as I can) to approve my ſelf to Conſcience and Omniſcience. For the Book no doubt, but Carping Momus *may pick ſome Criticiſmes out of it to ſnarl at; but I hope of no great moment;*

moment; indeed the beſt Actions of the beſt men are ſubject to Detraction, eſpecially their Writings. A detractione neminem eſſe immunem niſi qui nihil ſcripſerit, *ſays a Reverend Father of our Church;* Tho. Epiſc. Covent. *And it is well obſerved by another,* Facillimum quidem eſt, & fuit ſemper, alienum opus reprehendere; ſed non perinde facile, aliquid ſimile aut præſtantius conficere: *And all that I ſhall add, is that of the Poet,*

Carpere vel noli noſtra, vel ede tua.

Or as it may be Engliſhed,

Either Commend me, or come and mend me.

——————Si quid noviſti rectius iſtis,
Candidus imperti, ſi non, His utere mecum.

Vale, Vive bene beateq;

THE

THE PRÆFACE OR INTRODUCTION.

THE *LIBERAL ARTS* or *SCIENCES* are reckoned Seven, *Grammar*, *Rhetorick* and *Logick*, *Muſick*, *Arithmetick*, *Geometry*, and *Aſtronomy*. Theſe may not unfitly be divided into Two parts:

Firſt, The *Scholaſtical*, or Speaking Part, *Grammar*, *Rhetorick*, and *Logick*.

Secondly, The *Mathematical*, or Meaſuring Part, which comprehends *Arithmetick*, *Geometry*, and *Aſtronomy*.

Muſick which is uſually placed in the middle, partakes of a middle or mixt Nature betwixt theſe Two Generals. For *quoad ſonum* it may belong to the former, to wit, the *Speaking Part*, being an inarticulate kind of *Speaking*: But *quoad proportionem* to

the latter, to wit, the *Meaſuring Part*, conſiſting chiefly in due Meaſures and Quantities.

It is not my purpoſe at preſent to ſpeak any thing of thoſe which I call the *Speaking Sciences*, only to ſhew how proper this Diviſion is by their Definitions you may take them thus; *Grammatica eſt Ars rectè Loquendi, Rhetorica benè Loquendi, Logica benè Diſſerendi.*

The Poet diſtinguiſhes and commends them thus;

G. *Grata quidem Ratio eſt concordi voce relata.*

R. *Gratior eſt Ratio, veniens ratione venuſta.*

L. *Grata ter eſt Ratio veniens ratione probata.*

Of Grammers there is as great Variety as of Languages. *Latine* is a general Language in theſe Weſtern parts of the World, and the ground of moſt other Languages in theſe parts: ſo is *Greek* in the Eaſtern: *Hebrew* the Language we ſhall ſpeak in Heaven ſay ſome, and the Original Language of all others ſay moſt; And *High Dutch*, as Ancient as the Building of *Babel*, ſay many.

But

But of all other Grammars and Languages moſt ſutable to this preſent Subject, is that moſt incomparably ingenious Invention of Biſhop *Wilkins* (that Famous *Mathematician*) which he calls his *Philoſophical Language*, with a way of writing the ſame, called his *Univerſal Character*, which if it would be as Univerſally, and as induſtriouſly practiſed, as it is moſt excellently, and ingeniouſly contrived: By this means all perſons (at leaſt all of any competent capacity) of what Nation or Language ſoever might converſe and correſpond with one another as familiarly, as *Adam* and *Eve* in Paradiſe, or thoſe Eight in the Ark, when the whole World ſpoke but one and the ſelf-ſame Language. But enough of Theſe as to my Purpoſe (to wit) of *Grammar*, *Rhetorick*, and *Logick*.

And for *Muſick*, which I call the Meane in this Diviſion, I mean not to run any large Diviſions upon it; that Art being rather learn'd by Practice than by Precept; and my Aim being rather to propound Demonſtration to the Eye, than Muſick to the Ear. Indeed the Muſick of the Sphears might have been no improper Theam, but that being grounded upon the old erronious Opinion of their *ſolid Orbs*, are both diſclaimed long ſince by later Writers. Indeed, I read, that *Philo* the *Jew* ſaies, that hereby *Moſes* was inabled to tarry in the

Mount 40 days, and 40 Nights, without eating or drinking any thing, becauſe he heard the melody of the Heavens: and *Plutarch* ſpeaks as if a man might hear this *Muſick* if he were an Inhabitant in the Moon: But theſe conceits being by the Learned long ſince exploded, I ſhall no further touch upon this ſtring, *viz. Muſick*, with the other *Speaking Arts* before-mentioned: But paſſing by all theſe, That which I call the *Mathematical*, or *Meaſuring Part* of the *Liberal Sciences* is the ſubject I intend to treat upon; and (as I apprehend it) may fitly be comprehended in theſe Three, *Arithmetick*, *Geometry*, and *Aſtronomy*: *Aſtronomy* may be ſaid to meaſure the Heavens, *Geometry* the Earth, and both by means and help of *Arithmetick*. This laſt conſiſting chiefly in *The Rule of Three*, the two former in the Reſolution of a Triangle. And when I ſay *Geometry* meaſures the Earth, it is to be underſtood the whole *Globe* of the Earth, containing *Sea* and *Land*, with all other Bodies therein contained, whether *Superficies* or *Solids*: What concerns the Land, may in one word be called *Surveying*: What concerns the *Sea*, is properly called *Navigation*: And not only both theſe, but alſo *Aſtronomy* with theſe may be called the Taking of Heights and Diſtances: Surveying being Taking Heights and Diſtances at Land, Navigation at Sea,

Aſtro-

Aſtronomy in the Heavens, and of Aſtronomy, the moſt Mathematical and Practical part is Dialling, or finding the hour of the day by the Sun, and the hour of the night by the Moon and Stars.

Therefore to ſumm up all in ſhort, the Mathematical part of the Liberal Sciences are three, *viz. Arithmetick*, *Geometry*, and *Aſtronomy*. The Practical part of theſe are principally three, *Surveying*, *Navigation*, *Dialling*. Theſe three are performed by Trigonometrie, or the Reſolution of a Triangle, which derives its name from Three, and conſiſts of Three Angles, and Three Sides, and theſe found by the *Rule of Three*, makes good that old Motto, *Tria ſunt Omnia*.

You ſee then, That the grand Deſign of the Mathematicks is to take Heights and Diſtances at Sea, at Land, and in the Heavens, and to meaſure not only the Heavens and the Earth, but all other Bodies therein contained, whether Superficies or Solids. At Land, to find Heights by Diſtances, and Diſtances by Heights, where either is acceſſible; and both theſe by Angles of Poſition and Interſection of Lines, where both or either of them are inacceſſible. At Sea for Diſtances, to find the Diſtance of a Ship North and South, which is called Latitude, or its Diſtance Eaſt and Weſt, which is called Longitude, or the Diſtance

from any of theſe Cardinal points, which is called the Rhumb; Laſtly, the Diſtance Run upon the ſaid Rhumb, or upon ſuch a point of the Compaſs: I am not ignorant in ſtrict propriety of Terms, a Rhumb differs from a point of the Compaſs. A Rhumb containing two points, *viz.* the point and its oppoſite point: the former being the whole Diameter, the latter only the Semidiameter of the Compaſs from the Centre, ſo that there are (to ſpeak according to that nicetie) only 16 Rhumbs, and 32 points of the Compaſs; but for more plainneſs and eaſe to the Learner, I make them here ſignifie the ſame thing, being the ſame in ſubſtance. And by an Account kept of theſe to know the Diſtance of one Ship from another, or of the ſame Ship from the place from whence it ſet Sail, or from the place to which it is bound; and conſequently how to direct her courſe accordingly. In the Heavens, to find the Diſtances of the Sun, Moon, and Stars from one another, of one Star from another; particularly concerning the Sun, his Diſtance from the Equinoctial, which is called his Declination: His Diſtance from any of the 12 Signs which is called his place in the Zodiack: His Diſtance of Riſing and Setting from the due point of Eaſt or Weſt, which is called his Amplitude: His Diſtance at any time from any point of the Hori-

zon

zon, which is called his Azimuth; together with the Diſtances of all thoſe Cœleſtial Bodies, to wit the Planets, the Sun, Moon, and fixed Stars from this Terreſtrial Globe: All theſe may be called Diſtances in Aſtronomy: And for Heights, both in Aſtronomy and Navigation, they are the ſame; as taking the Height or Altitude of the Sun and Stars at any time above the Horizon which is called their Almicantaras, particularly the Meridian Altitude, either of the Sun or Stars: And thereby to find the Height or Elevation of the Pole: So that by all this you ſee, that the taking of Heights and Diſtances at Sea, at Land, and in the Heavens is the moſt principal, and the moſt practical part of the Mathematicks: And theſe are all perform'd by Trigonometry, or by the making and meaſuring, the framing and reſolving of a Triangle, either Plain or Sphærical: Suppoſing theſe Heights and Diſtances to be the ſeveral parts, that is, either the ſides or Angles of a Triangle. And to find by ſome of theſe Parts known, other Parts unknown or *per Data, Queſita.*

This Reſolving of a Triangle, or Trigonometry, reſolves all Mathematical Queſtions, and may be performed by Inſtruments and Protraction Geometrically: But principally by the Rule of Proportion Arithmetically; worthily carrying the name

of the *Golden Rule*, for the admirable Effects it produceth, especially in these Mathematical Operations. And for the fuller and more ample Explanation of these particulars, I shall proceed to Discourse of them in this Method in 3 Books.

I. In the First I shall Treat of Arithmetick, under the several kinds of it, *viz. Common*, *Decimal*, *Logarithmetical*, and *Instrumental*. And in the sequel of the Discourse, as occasion shall be, I shall shew how these may be applyed to the Resolution of a Triangle, or any other Mathematical Question.

II. In the Second Book I shall Treat of Geometry, or any other Mathematical Question. First in general, and of the Three Famous Grametrical Figures, the *Circle*, *Square*, and *Triangle*; especially the last of these. And then Secondly, In particular I shall speak of Geometry, as it consists in Surveying and Navigation; and shew that these are no more than the Resolution of a plain Triangle.

III. In the third and last Book I shall Treat of Astronomy, and therein of Projection of the Sphære, Questions of the Sphære and Dialling, and shew that as Surveying and Navigation are performed by the Resolution of a Plain Triangle, so these by the resolution of a Sphærical Triangle.

THE

The First BOOK.

OF ARITHMETICK.

CHAP. I.

Which as before I do divide into 4 *Parts*, 1. Common, 2. Decimal, 3. Logarithmetical, 4. Instrumental, *or* Lineal. *Of these in their order.*

COMMON ARITHMETICK to those that are expert in it, especially in working by Fractions, is of singular use, and absolutely necessary; and for the most part of sufficiency enough for Mathematical Operations; and withal it is to be supposed, That no man that attempts the Mathematicks, but in some measure understands those Common Things of *Numeration*, *Addition*,

Subſtraction, *Multiplication*, and *Diviſion*, therefore I ſhall be very brief in them; and only tell you

SECT. I.

Numeration is the Reading of Figures according to the value they receive from the place they ſtand in: The Firſt place ſignifying Units: the Second, Tens: the Third, Hundreds: And the Fourth, Thouſands, according to the enſuing Table.

Numeration

Numeration TABLE.

Units.	1
Tenns.	12
Hundreds.	123
Thouſands.	1234
X. Thouſands.	12345
C. Thouſands.	123456
Millions.	1234567
X. Millions.	12345678
C. Millions.	123456789
Thouſand Millions.	1234567891
X. Thouſ. Millions.	12345678912
C. Thouſ. Millions.	123456789123
Dillions.	1234567891234
X. Dillions.	12345678912345
C. Dillions.	123456789123456
Thouſand Dillions.	1234567891234567
X. Thouſ. Dillions.	12345678912345678
C. Thouſ. Dillions.	123456789123456789
Trollions.	1234567891234567891
X. Trollions.	123456789 12345678912
C. Trollions.	1234567891234567891 23
Thouſand Trollions	12345678912345678912 34
X Thouſ. Trollions.	123456789123456789123 45
C. Thouſ. Trollions	1234567891234567891234 56

One Hundred Twenty Three Thouſand Four Hundred Fifty Six Trollions Seven Hundred Eighty Nine Thouſand One Hundred Twenty Three Dillions Four Hundred Fifty Six Thouſand, Seven Hundred Eighty Nine Millions One Hundred Twenty Three Thouſand Four Hundred Fifty Six.

SECT.

SECT. 2.

II. *Addition* is the adding of two or more Sums together, to find the Total.

In like Denominations.		In divers Denom.		
		D.	M.	Sec.
6789432	Degrees D.	136	11	9
1903467	Minutes ′	320	10	45
8542079	Seconds ″	429	54	26
17234978	Summa Total.	886	16	20

SECT. 3.

III. *Subtraction* is the Substracting or Deducting of one Sum from another, to find the Remainder.

	D.	′.	″.
As from	2345	36	49
Take	873	18	13
Remainder is	1472	18	36

SECT. 4.

IV. *Multiplication* is a Compendious *Addition.*

The TABLE.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	30	90	100

Find the Multiplicand in the Top of the TABLE, and the Multiplyar in the side thereof, and in the Square where These meet, is the Product of the Nine Digits; as 7 times 9 is 63, and so of the rest.

In Multiplication the Sum to be Multiplyed is called the Multiplicand, the Sum by which the other is Multiplyed, is called the Multiplyer, and the Number produced by these is called the Product.

Multipli-

Multiplicand	463298765
Multiplyer	23
	1389896295
	926597530
Product	10655871593

To Multiply compendiously by any of the Nine Digits, without charging the Memory.

To Multiply any Number by 2,
Either double it, or set it down twice, and add the two sums together.

To Multiply by 3.
To the Number given, add the double thereof, the sum of them shall be the Product.

To Multiply by 4.
Double the Duplication of the Sum given.

To Multiply by 5.
To the Number given add a Cypher, and the half of that Number is the Product.

To Multiply by 6,
To the Number given add a Cypher, then take the half of that Number, and to it

it add the Number given, the Sum of them ſhall be the Product.

To Multiply by 7,

To the Number given add a Cypher, and take the half thereof, to which half add the double of the Number given, the Sum ſhall be the Product.

To Multiply by 8,

To the given Number add a Cypher, and from thence ſubſtract the double of the Number given, the Remainder will be the Product.

To Multiply by 9,

Add a Cypher to the given Number, and from thence ſubſtract the given Number, the Remainder ſhall be the Product.

To Multiply by 10 add a Cypher, to multiply by 100 add two Cyphers, by 1000 add three Cyphers, and it is done.

If Cyphers be in the middle of the Multiplyer among other Figures, remove the next Figure a place farther where any Cypher comes, and add the Lines together in the ſame order they ſtand.

SECT.

SECT. 5.

Division is a compendious Substraction, wherein the Sum to be divided is called the *Dividend*; the Sum by which we divide the same, is called the *Divisor*; what is produced thereby, is called the *Quotient* and the *Fraction*. *Example.*

```
                 2x(1
                 xoox(8
Dividend  4679x38|203440 18/23 parts.
Divisor   2333333 Quot. Fraction.
          22222
```

In dividing any great Sum, to know certainly what Figure to put in the Quotient, and to Divide the Sum by Substraction instead of Multiplication.

First, Set down your Divisor, and against it the Digit 1, In the next place double it, and against that set the Number 2. Add the former two Sums together, and against that set the Figure 3, to that Number add the Divisor, and against that set the Figure 4, *&c.* so you will have a Table of the 9 Digits, and may see against every Figure how much the Multiplication of your Divisor by any Digit will amount unto; so that when you see your Remainder in

in your Diviſion, look in the Table for the neareſt Number leſs than your Remainder, and that Digit which ſtands againſt that Number, you muſt alwaies put in your Quotient.

To Divide by any Number that hath Cyphers to the Right Hand.

To Divide by 10, cut off the laſt Figure towards the Right Hand, and the reſt of the Figures are the Quotient. To Divide by 100 cut off two of the laſt Figures, by 1000 three, *&c.* To Divide by 20; cut off the laſt Figure, and take the half of the Remainder.

As for Example.

To know how many Pounds there is in 243L 6 Shillings is the ſame as to Divide by 20. Therefore cut off the laſt Figure 6, and take half the reſt, that is, half 243, is 121 Pounds, and there remains 1, and the 6 cut off for the Fraction, which is $\frac{16}{20}$, or 16 ſhillings, and ſo in any other Sum. The ſame Reaſon holds if you be to divide by 30, cut off the laſt Figure, and take the third part; and if by 40 the Fourth, 50 the Fifth part, and ſo forwards. The remaining Fraction being alwaies the parts of its reſpective Integer.

These Four, *viz.* *Addition*, *Subtraction*, *Multiplication*, and *Division* (not to mention *Numeration*, which is only the Reading of Figures) do contain the whole Practice of Arithmetick, being judiciously applyed and made use of according to the Nature of each several Question, and by the Invention of Logarithmes these Four are reduced to these Two only, *viz.* *Addition* and *Substraction*, as you will hear hereafter.

But besides these Essential parts of *Arithmetick*, there are divers other necessary Rules for directing the way or manner of Operation by these.

SECT. 6.

Therefore in the sixth place is that incomparable *Rule of Three*, worthily carrying the Name of the *Golden Rule*, especially, and above the rest to be taken notice of; which yet in the Operative and Practick part of it, is no more but the right Application of Multiplication and Division, that is, by Three Numbers given, to find a Fourth, according to these Two Directions.

First, If the Third Number be greater than the First, and the Fourth required to be greater than the Second, then Multiply the Second by the Third, and Divide the Pro-

Product by the First, the Quotient gives the Fourth Number.

As if 12 *yds.* of Cloth cost 3 *l.* what cost 435 *yds.*

3	xz(9
1305	x3ø 8[108 9/12 or 3/4 *l.*
	xzzz
	xx

Secondly, If the Third Number be greater than the First, and the Fourth required to be less than the Second, then Multiply the Second by the First, and Divide by the Third. As,

If 12 men raise a Fort in 8 days, in how many days shall 24 men do it.

8	xø
12	9ø[4 *dayes.*
96	24

I. The Direct Rule is, when more requires more, and less less.

II. The Back Rule is, when more requires less, or less more.

For the placing the Numbers right, observe this Rule in the *Direct Rule of Three.*

Three Numbers being given, the Question is annexed but to One, and this must alwaies

alwaies be the Third Number, and that which agrees with this Third in the same Denomination must be the First, and that which remains the Second. As if the Question were put, If 10 yards cost 8 *l.* how many yards may we buy for 12 *l.* place them thus; If 8 buy 10, what will 12 buy.

SECT. 7.

The next Rule which I shall here mention, is the *Rule of Fellowship*, which is, when several Merchants or other Men put in several Sums together into Stock, and Trading therewith, they either gain or lose so much in the whole; Now the Question will then be, what every particular Adventurer should have of the Profit, or bear of the Loss according to each mans due proportion. To do this, say, If the whole Stock produce so much as is the whole Profit or Loss, then what gives the First mans, Second and Third mans part of Stock severally making each mans particular Stock the Third Number to these other Two Numbers, (*viz.* the whole Stock, and the whole Profit or Loss) and the Fourth will be each particular mans Profit and Loss.

As if Three Merchants put into Stock 1684 pounds, The First puts in 750 *l*, the Second 620 *l*, and the Third 314 *l*, They

Trade

Trade together till they gain 500 *l.* The Queſtion is what every one ought to have for his ſhare.

For the Firſt ſay,
If 1684 *l.* give 500 *l.* what gives 750 *l.*

For the Second ſay,
If 1684 *l.* give 500 *l.* what gives 620 *l.*

For the Third ſay,
If 1684 *l.* give 500 *l.* what gives 314 *l.*

SECT. 8.

To find a mean proportional, Multiply one Extream Number by the other, and the Square Root of the Product is the mean proportional: As 72 multiplyed by 32, the Product is 2304, the Square Root whereof is 48: For proof 48 multiplyed by 48, produces the ſame Number 2304, and this is the mean proportional betwixt 72 and 32.

SECT. 9.

To conclude this particular, The Extracting the Square and Cube Root is the hardeſt Leſſon in *Common Arithmetick*: Thoſe which have a mind to do them this way, I ſhall refer them to the Sea-mans *Kalender*, in the latter end of which Book it is plainly

ſet

ſet down: But this way being very difficult and tedious, I ſhall ſhew you very ſhort and eaſie wayes hereafter to do it by *Logarithmes*, *Gunters Sector*, *VVingates Rule of Proportion*, or any ordinary Line of Numbers, whereunto I refer the Reader at this time, and ſhall proceed to the Rule of Falſehood.

SECT. 10.

The *Rule of Falſehood* is ſo called, becauſe it teacheth (by feigned Numbers taken at all adventures) to find out the true Number that is demanded; And that 1. Either by one Falſe Poſition: Or, 2. When it requires two falſe Poſitions.

The Rule of one falſe Poſition is this, Inſtead of 3 Numbers given in the *Rule of Three*, by thoſe to find a fourth; In this Rule we have but one Number that cometh in uſe to work by: Unto the likeneſs whereof we muſt feign or deviſe two other Numbers, which ſhall hold the ſame proportion to one another which the third number given ſhall have to the number ſought As for Example. I delivered at Intereſt a certain Sum of Money at 6 *per Cent.* and at the end of 10 years I received 500 *l.* for Principal and Intereſt, I demand how much was the Principal Sum?

To anſwer this, let us feign a Number at pleaſure, and with the ſame, let us make

our

our Diſcourſe even as if it were the principal Sum we ſeek for: As Suppoſe I delivered him at the firſt 200 *l*, which in 10 years at *6 per Cent. Simple Intereſt* will amount to 120 *l*, which added to the principal 200 *l*, makes together 320 *l*, but this ſhould be 500 *l*: Therefore ſay by the *Rule of Three*, If 320 *l*. come of 200 *l*, what doth 500 *l*. come off? Multiply the two laſt Numbers 200 and 500 *l*. by one another, they amount to 10000 *l*, which divide by the firſt Number 320, and the Quotient is 312½, which is the Sum ſought for, or the Principal Sum delivered at Intereſt at the firſt. You will find the ſame number, If you ſuppoſe 160 *l*. to come of 100 *l*. then 500 *l*. will in like manner come of 312½ as before: Or any other ſuppoſed Numbers which hold the ſame proportion to one another, which 500 *l*. holds to the Number ſought.

2. The Rule of Two Falſe Poſitions I ſhall endeavour to make plain on this manner, ſuppoſe an 100 Ducates is to be divided amongſt 3 Perſons, the firſt to have a certain number of them, the ſecond muſt have twice ſo many as the firſt, abating 8 Ducates, and the third muſt have 3 times ſo many as the firſt, leſs by 15 Ducates. Now I demand how many every one ſhould have.

Firſt, You may imagine any number at pleaſure which you ſhall call the firſt Poſi-

tion,

tion, and with the ſame you ſhall work inſtead of the true Number; and if you ſee you have miſſed of the true Number that you ſeek, then is the laſt number of the work either too great, or too little; which number you ſhall note with the ſign of more or leſs, and that you call the firſt Error; now the ſign of more ſhall be noted with this Figure +, the ſign of leſs with this plain Line −. For Example, Suppoſe the firſt man in the Queſtion had 30 Ducats, then by the order of the Queſtion, the ſecond ſhould have 52, the third 75; theſe three added, make 157, and I ſhould have but 100; ſo that this firſt Error is too much by 57, and therefore I ſet down the firſt Poſition 30 with his Error 57 thus, 30+57.

The next thing to be done is to ſuppoſe another Number, which you muſt call your Second Poſition, and note its exceſs or want of the true number, and that you call the ſecond Error. As for Example, Suppoſe you take 24 for the firſt Man in the Queſtion, then by the order of the Queſtion, the ſecond was to have 40, and the third 57; theſe three added make 121, and I muſt but have 100, ſo the ſecond Error is too much by 21, and therefore I note it down under the former thus, $\left\{\begin{matrix}30+57\\24+21\end{matrix}\right\}$ Then you

you muſt Multiply the firſt Poſition 30 by the ſecond Error 21 Croſswiſe; and again the ſecond Poſition 24 by the firſt Error 57, (and this muſt alwaies be obſerved) and note down the two Products, *viz.* 630, and 1368, then obſerve this Rule.

Both ſigns alike Subſtraction *do require,*
But unlike ſigns Addition *will deſire.*

And accordingly, If both ſigns be alike, that is, both too much, or both too little, you muſt ſubſtract the leſſer Product from the greater; as in this Example, 630 from 1368, and note the Remainder, which is 738, and this muſt be your Dividend. Again, you muſt ſubſtract the leſſer Error from the greater, *viz.* 21 from 57 Reſts 36, this muſt be your Diviſor. Therefore Divide 738 by 36, the Quotient ſhall be the true Number ſought, or the Number of Ducats which the firſt man was to have, *viz.* $20\frac{1}{2}$, and conſequently the ſecond man had 33, and the third $46\frac{1}{2}$, making in the whole 100.

But note, that if the ſigns of the two Errors had been unlike, that is, if one of the Errors had been too much, and the other Error too little, then inſtead of ſubſtracting the one Product from the other, you muſt have added the one to the other; and likewiſe you muſt have added both the

Errors together, and by the Sum of those Errors you must have divided the Total Sum of both the Products, and the Quotient should then have been the true Number sought. As for *Example.*

If the two Positions were 24 and 20, you will find the first Error will be 21 too much, and the second will be 3 too little. Therefore multiply 24 by 3, produceth 72. Likewise, multiply 20 by 21 produceth 420, and because the signs of the Errors are unlike, add them 492, which shall be your Dividend; and again add the lesser Error 3 with the greater 21, makes 24 for your Divisor; and lastly, Divide 492 by 24, the Quotient is 20½ as before, for the first mans share; and consequently the second must have 33, and the third 46½, making up together 100, as in the former *Example*: And this I hope is sufficient for the understanding of this excellent *Rule of Falsehood*, or of *False Positions*, in the several kinds of it; though I might have added variety of other *Examples*: But to avoid prolixitie, and for brevities sake these may suffice.

CHAP.

CHAP. II.

Of Decimal Arithmetick.

IN the Second place moſt Mathematicians do uſe and extol *Decimal Arithmetick*, and do accommodate all their Scales, Rules, and Lines to this kind of *Diviſion*, and not without cauſe, whereby they have this advantage, that the Fractions which in Common Arithmetick are of divers Denominations, in this are all Decimal Fractions; that is, $\frac{1}{10}$ $\frac{1}{100}$ $\frac{1}{1000}$ parts of any thing, and by adding Cyphers, they may bring them to ſmaller Fractions, and this makes the working by Fractions far more eaſie and exact.

To Reduce ordinary Fractions into Decimal.

Multiply the Numerator (or the Number above the line) by 100, or 1000, and Divide the Product by the Denominator (or that Number which is under the Line) and the Quotient ſhews the Numerator of the Decimal Fraction; As $\frac{3}{4}$ multiply 3 by 100 (by adding 2 Cyphers) makes 300, Divide 300 by 4, and the Quotient will be 75, ſo

that $\frac{3}{4}$ in Common Fractions is $\frac{75}{100}$ in Decimal.

CHAP. III.

Of Logarithmetical Arithmetick.

IN the third place the Invention of *Logarithmes* by my Lord *Napier* is an excellent Invention; for what is done by Common or Decimal Arithmetick by Multiplication and Diviſion, are done by theſe artificial Numbers, by *Addition* and *Subſtraction* only: The Square Root is extracted by Bipartition, and the Cube Root by Tripartition; to explain theſe more particularly in all the neceſſary parts of Arithmetick, as firſt Numeration in Logarithmes, or how to read them.

SECT. I.

In the Table of Logarithmes you will find againſt every Natural number (from 1 to 1000) its artificial Number or Logarithme; ſo if you ſeek the Logarithme of 50, is 1.698970, and if you would find the natural number to this Logarithme 1.62249, over againſt it you find 42, and ſo the like

of

of all others. And what is ſaid of Logarithmes, as to numbers, the like is to be underſtood all along of Natural Sines and Tangents, and their Artificial Logarithmes, as you may perceive by the Tables.

By the Table of Logarithmes, to find the Logarithme of a Number conſiſting of 4 places, as ſuppoſe 5628, *the ordinary Numbers reaching only to* 1000.

Firſt, Take the three firſt Figures, *viz.* 562, and find that in the firſt Colum under the letter *N*, then for the laſt Figure, which is 8, Find amongſt the great Figures in the head of the Table, and in the common Area or meeting of theſe two lines you will find .750354, before which I place 3 for the proper Characteriſtick makes 3.750354 the Logarithme of 5628, which was required.

Now beſides the Logarithme it ſelf, there is alſo to be placed before it his proper Characteriſtick, *viz.* a Figure conſiſting of an Unite leſs than the number of digits or places in the ſaid number, whoſe Logarithme is to be expreſſed: As the Characteriſtick of any number conſiſting of one Figure is 0, of two Figures 1, of three Figures 2, *&c.* as the Logarithme of 415 is .618048, and becauſe it conſiſts of three Figures, its Cha-

racteriſtick is 2, placed thus with a Comma 2,618048.

To find the Logarithme of a proper Fraction, as of $\frac{2}{3}$. Firſt I find the Logarithme of its Denominator 3 to be 0,477121, then I find the Logarithme of its Numerator 2 to be 0,301030: And ſubſtracting the latter from the former, there remains the Logarithme of $\frac{2}{3}$, which is 0,176091.

SECT. 2.

To Multiply by Logarithmes is to add one Logarithme to another, and that does the ſame as to multiply the natural Numbers: As to Multiply

24	by	Log.	1.380211
32	is	Log.	1.505150
48			2.885361
72			
768			

Over againſt which Logarithme you will find in the Table the natural number 768

SECT. 3.

To Divide, is to Subſtract one Logarithme from another, and that does the ſame

ſame as to divide one natural number by another.

As to divide	36 whoſe Log.	1.556302
by	12 whoſe Log.	1.079181
is	3 Reſts	0.477121

Which Logarithme you will find to anſwer to the natural Number 3, the Quotient as before.

SECT. 4.

To Perform the *Rule of Three* by Logarithmes. As in *Common Arithmetick* we multiply the Second Number by the Third, and divide by the firſt (which is the *Direct Rule of Three*) ſo you muſt add the Logarithmes of the ſaid Second Number and Third together, and from the Total ſubſtract the Logarithme of the firſt number, and the remainder is the fourth number ſought. As for *Example.*

If 12 give 4, what gives 60.

60

(0 Log. of 4 is 0.602060
240 of 60 is 1.778151
x22
x Added are 2.380211
20 Subst. the third Numbers Log. 1.079181
Rests 1.301030

Which in the Table you will find to be the Logarithme of 20, the Quotient by Common Arithmetick as above.

SECT. 5.

To Extract the Square Root (which is the same as to find a mean proportional between 1 and the numbers given) therefore to do this observe, That half the Logarithme of the Number given is the full Logarithme of the Square Root: So the Log. of 144 is 2.158362, half whereof is the Square Root, *viz.* the Log. of 12, 1.07918.

SECT. 6.

To Extract the Cube Root is likewise no more but to take the third part of the Loga-

Logarithme of any Number, whoſe Cube Root is demanded, which third part is the Logarithme of the ſame Cube Root: As for *Example* The Log. of 64 is 1.806180, the third part whereof is 0.602060, which you will find to be the Logarithme of 4, which is the Cube Root of 64; the reaſon is, becauſe the Cubiq; Root is always the firſt of two mean proportionals betwixt 1, and the number given:

Note that any Number multiplyed into it ſelf, the Product is the ſquare number, and the Square Root is that ſame firſt Number; as 8 multiplyed by 8 gives 64 his Square, and his Root 8. And multiply a Square Number by his Root gives you the Cube; as 64 multiplyed by 8 gives 512 the Cube Number, his Root 8.

SECT. 8.

To find a mean proportional between two Extream numbers given. Add the Logarithmes of the two Extream numbers together; the half of the Sum ſhall be the Logarithme of the mean proportional required. As to find a mean proportionall betwixt 8 and 32, which ſhall bear the ſame proportion to 32, as 8 does to it.

The Logar. of 8 is	0.903090
The Logar, of 32 is	1.505150
Added together is	2.408240
Half whereof is	1.204120

The Log. of 16, the mean proportional; for as 16 is to 8, so is 32 to 16.

Thus you have Numeration, Multiplication, and Division by Logarithmes: Also the Rule of Three, The Extraction of the Square and Cube Root, and to find a mean proportional between two Extream Numbers given.

He that desires to know more of these, or more at large concerning these, I refer him to the latter end of *Gunters* works, from whence these are for the most part collected.

CHAP. IV.

Of Instrumental Arithmetick.

IN the Fourth place I proceed to that kind of Arithmetick, which I call Instrumental or Lineal, because it is wrought upon Lines and Mathematical Instruments. Those which I shall mention are these Three, Mr. *Gunters*

Gunters Sector, Mr. *Wingates* Rule of Proportion, and Mr. *Seth Partridge*'s Sliding Rule, being only the Lines of Numbers, Sines, and Tangents diverſly improved.

SECT. 1.

The Excellency of *Partridge*'s Sliding Rule conſiſts in this, That one may work upon it without Compaſſes, and after the ſame be right ſet, it often reſolves ſeveral queſtions at once, without altering the poſition; only thoſe which I have ſeen, are ſet to ſo ſhort a Radius, which makes it leſs exact, and more ſubject to Error. I ſhould adviſe the Reader to buy the Book, as being very plain, and for the accommodating of it to any other Lines of Numbers, Sines, and Tangents, which may be to a larger Radius, and to uſe Compaſſes, obſerve this general Rule, That what is Termed in the uſe of that Inſtrument, [Set ſuch a number on the Firſt to another number on the Second, Then againſt the Third Number on the Firſt, is the Fourth on the Second.] The ſame upon the plain lines of Numbers, Sines and Tangents, with a pair of Compaſſes, is thus worded [Extend the Compaſſes from ſuch a number to ſuch a number, (or from the firſt number to the ſecond) The ſame Extent the ſame way will reach from the Third Number to a fourth required. This

difference obſerved, the ſame Book and Directions will ſerve for either this Rule, or any other line of Numbers, Sines, and Tangents whatſoever.

SECT 2.

Next as to Mr. *VVingates* Rule of Proportion, its excellency conſiſts in this, that it is done to a very large Radius, and very neat diviſions, and therefore ſo much the more exact.

Likewiſe by this you have the Square and Cube Roots by inſpection only, and the reaſon is, becauſe to a very large Line of Numbers, called by him the great Line of Numbers there is adjoined a line of Numbers twice repeated to find the Square Root and mean proportions, called by him the mean line of Numbers, and likewiſe a line of Numbers 3 times repeated, called his little line of Numbers, and this applyed to the aforeſaid great line of Numbers finds the Cube Roots, and both theſe by inſpection only.

In other principal Reſpects both theſe Rules do agree with the lines of Numbers, Sines, and Tangents upon the outward edge of *Gunters* Sector, he being the firſt Author and Inventor thereof; therefore having hinted to you wherein the particular Excellencies of theſe other two Rules do con-

conſiſt, I ſhall ſhew you the general Uſe of theſe Lines of Numbers, Sines and Tangents upon the Sector (being the ſame in ſubſtance with the two former, only differently improved) as I ſaid before.

SECT. 3.

Therefore in the third place *Gunters* Sector I account the moſt excellent Inſtrument of all other, and moſt general for all Uſes, Intents, and Purpoſes whatſoever; not only in working all proportions, but it may alſo eaſily be improved to an excellent Quadrant and Theodolite, thereby to take Heights and Diſtances, and by that means to become a *Panorganon*, an Univerſal Inſtrument to do any thing or every thing both for Calculation and Obſervation: But in this place I intend only to inſiſt upon what it hath Common with *Partridge*'s Sliding Rule, and *Wingate*'s Rule of Proportion: To wit,

The Lines of Numbers, Sines, and Tangents.

Then to ſhew you,

I. *Numeration* upon the Lines or the reading of them, which is on this manner. The Line of Numbers conſiſts uſually of two parts, the former and the latter, each numbred with theſe Figures, 1.2.3.4.5.6.7.8.9.1. being the ſame Line twice repeated.

peated. Now these Figures according as you have occasion, may signifie either single Units, Tens, Hundreds, Thousands, Ten Thousands, or any other imaginable Number, by Mr. *Wingate* called Primes, Tens, Centesmes, Millains, *&c.* And the smaller divisions signifie proportionably in order to what their Primes are taken for. And in reference to the first and second part of this Line take this Rule. If the Primes that is the large Divisions where the Figures are plac't do in the former part signifie Units, only the subdivisions are $\frac{1}{10}$ Tenths parts of the same Units, *&c.* accordingly if smaller Divisions happen under these: And the Primes in the second or latter part of the Line signifie Tens to the former Units, and Hundreds, if the first Primes signifie Tens, Thousands, if the first signifie Hundreds, *&c.* Now to give one Example for all, The first Prime in the second part is divided first in 10 parts, and those Tens are again really divided into Ten more, and these last at least supposed to be divided into Ten more still; so that suppose a man would number upon the Line 11575. The first part of the Line signifies the First Figure 10 Thousand. The first Prime in the second part signifies the second Figure 1000. Five of the larger Divisions into Tens signifie 500, the seven subdivisions

diviſions into other Tens, ſignifie 70, and adding to theſe half of the next ſmall Diviſion by Eſtimation, ſignifies the odd 5, ſo at that very place you have 11575, and ſo of any other Number, whether bigger or leſſer.

For the Line of ſines, 'tis eaſie being numbred by 10. 20. 30. 40. 50. 60. 70. 80. 90. and the former part with 1. 2. 3. 4. 5. 6. 7. 8. 9. Theſe ſignifying ſingle Degrees, and the ſubdiviſions firſt into 6, each ſignify 10 Minutes, and thoſe again divided into halfs, which ſignifie 5 Minutes, the reſt ſupplyed by Eſtimation or Suppoſition: In the ſecond part they are firſt divided into Ten larger parts, which ſignifie each one Degree, and thoſe Ten ſubdivided into 6, which ſignifie each 10 Minutes, *&c.*

And what is ſaid of the Line of Sines, the ſame may be ſaid of the Line of Tangents, ſignifying alſo Degrees and Minutes, and proceeding forwards to 45, at the end of the Line, and ſo backwards to 90, where you begun, as you will ſee ſignified by Figures to that purpoſe.

Having as plainly as I can expreſſed *Numeration* upon the Rule, in the next place I come to ſhew you the next conſiderable Rule in *Arithmetick*, upon theſe Lines: To wit,

II. *Mul-*

II. *Multiplication*: And to Multiply, Extend the Compaſſes upon the Line of Numbers from 1 to the Multiplicator. This done, the ſame Extent the ſame way will exactly reach from the Multiplicand to the Product required: As for *Example*, To multiply 42 by 33, extend your Compaſſes from 1 to 33, the ſame Extent will reach from 42 to 1386; ſo to multiply 18 by 4, Extend the Compaſſes from 1 to 4 in the former part of the Line, and the ſame Extent in the ſecond part will reach from 18 to 72. The like is to be underſtood of all other Queſtions in Multiplication: Only take this other *Example*.

In Lands and Houſes ſold at ſo many years purchaſe. To find what the value of the whole will be by the Line of Numbers. The Extent of the Compaſſes from one to the number of years, will reach the ſame way from the yearly Rent, to the Sum of the Purchaſe. As the Extent from 1 to 20 will reach from 10 to 200, which ſhews, that 10 *l. per Annum* at 20 years purchaſe, is worth 200 *l.* and ſo of any other.

III. *Diviſion*: To Divide by the Lines, Extend your Compaſſes upon the Line of Numbers from the Diviſor to One: This done, if you apply the ſame Extent the ſame way from the Dividend, the move able point will fall upon the Quotient. As to

Divide

Divide 12 by 3, Extend the Compaſſes from 3 to 1, the ſame Extent will reach the ſame way from 12 to 4, which is the Quotient.

IV. To Perform the *Rule of Three*, or Three numbers being given to find a Fourth, Extend the Compaſſes from the Firſt to the Second, and that Extent the ſame way will reach from the Third number to the Fourth. As to take the ſame Example as in Logarithmes: If 12 give 4, what gives 60. Extend the Compaſſes from 12 to 4, the ſame Extent the ſame way will reach from 60 to 20, the number required.

V. To Extract the *Square Root* is the ſame as to find a mean proportional betwixt 1, and the number given; therefore divide the ſaid diſtance betwixt 1 and the number given into two equal parts, and at that point you will find the Square Root. As for *Example*, the middle diſtance betwixt 1 and 9, you will find to be at 3, which is the Square Root of 9, for 3 times 3 is 9.

Now by Mr. Wingate's *Rule to find the Square Root by inſpection only, take this Rule.*

When the Figures of the Number given are even, as when the Number conſiſts of 2 4 6 8 Figures, look the ſame number in the firſt part of the mean Line of Numbers,

and

and juſt over againſt it at the ſame point upon the great Line of Numbers (or indeed upon that Line of Numbers beginning in the middle, for this together with the other makes but up the great Line of Numbers compleat) there you have the Square Root by inſpection only; as 144 I find at the point *S*, and over againſt it 12, which is the Square Root thereof.

VI. To Extract the *Cube Root* is to find two mean proportionals betwixt the Number given, and 1: Therefore upon the Line of Numbers divide the diſtance betwixt the number given, and 1 into three equal parts, and the firſt of thoſe 3 parts from 1 is the Cube Root. For *Example*, Extend your Compaſſes from 1 to 4, and that will divide the diſtance betwixt 1 and 64 into three equal parts, therefore 4 is the Cube Root of 64, and to prove it, multiply 4 by 4 makes 16, and 16 multiplyed again by 4 makes 64, the Cube number, its Root 4.

Now to do this upon Mr. Wingate's *Rule, by Inſpection only, Take this Rule.*

When the Number propounded conſiſts of theſe Figures, *viz.* of 1. 4. or 7. find it in the firſt part of that which he calls the little Line of Numbers (being the Line of

Num-

Numbers three times repeated.) If it consiſt of 2 5 or 8 Figures, then find it in the ſecond part, or if of 3 6 9, then find it in the third part: And this done, obſerve, That at the very ſame point, upon the great Line of Numbers, you ſhall find the Cube Root you look for. As for *Example*, To find the Cube Root of 64, becauſe the number conſiſts of 2 Figures, therefore look for 64 in the ſecond part of the little Line of Numbers, and over againſt it exactly in the great Line of numbers you find 4, which is its Cube Root.

VII. To find a Mean Proportional betwixt two Extream Numbers, is only to divide the ſpace betwixt the Extream numbers into two equal parts, or by *Wingates* Rule, Extend the Compaſſes upon the mean Line of numbers, from one of the Extream Numbers to the other. This done, the ſame Extent applyed upon the great Line of numbers from either of the numbers towards the other, the moveable point will fall in the middle betwixt them; *viz.* upon the point repreſenting the mean proportional required. *Example*, 8 16 32.

Thus you have all the neceſſary parts of Arithmetick performed by the Lines of Numbers, Sines, and Tangents, and what is ſaid as to Numbers only, the like will hold as to Sines and Tangents alone, or theſe ſeverals

verals mixt, as between Sines and Numbers, Tangents and Numbers, or Sines and Tangents; as the nature of the queſtion ſhall require. He that deſires to know more concerning theſe, I ſhould adviſe to buy Mr. *Wingates* Book, called his Rule of Proportion, wherein you have the Rule it ſelf in paper, which you may get paſted upon wood; and will be no great coſt, and will ſhew you the whole Myſtery of it; Alſo I ſhould adviſe to buy *Seth Partridges*'s Book, ſhewing the uſe of his Sliding Rule, being plainer than *Wingates*, only obſerving the Rule before given for applying the Queſtions, ſo as to be wrought upon a plain Line of Numbers, Sines, and Tangents, and a pair of Compaſſes.

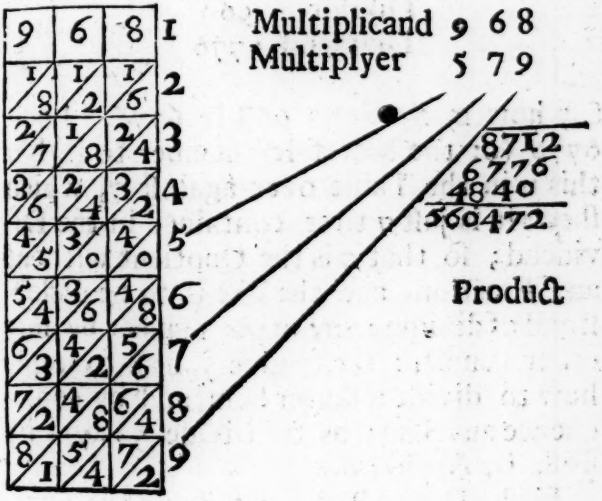
SECT. 4.

To conclude what may be ſaid in reference to *Inſtrumental Arithmetick*, I think it not amiſs to give you a ſhort Account of *Napiers Bones*, whereby Multiplication and Diviſion in *Common Arithmetick* is performed by Addition and Subſtraction only without charging the Memory.

Napiers Bones then are nothing elſe but *Pythagoras* Table with divided Lines. For Reading the Bones begin alwaies on the right hand, and take them as they lye in each Diagonal Line, and if there be two Figures

Figures in the ſame Diagonal, make but one of them by adding them together; and if they exceed 10, take 1 to the Figure of the next Diagonal immediately following. As for *Example.*

Againſt 9 you find in the firſt Diagonal 2 by it ſelf; in the next 7 and 4 which makes 11; therefore ſet down 1, and carry one to the next Diagonal, which is 5 and 1, and 1 you carryed makes 7, and in the laſt Diagonal ſtands 8 by it ſelf, ſo they are to be read 8712, and ſo of any other number whatſoever.



In

In Multiplication place alwaies the Multiplicand in the Top of the Bones, and against every Unite of the Multiplyer in the ſide, you will find the particular Product of the ſame Unite, which added together, as in the *Example*, makes up the whole Product as before.

For *Diviſion* place the Diviſor in the top of the Bones, and by finding the neareſt leſs number to the Dividend, you may find how often the Diviſor is contained therein.

As for Example.

Diviſor 968
Dividend 6776

Say how many times 968 in 6776. Look 6776 (or the neareſt leſs number to it) but this is in the Table over againſt 7, which ſhews it is juſt 7 times contained in the Dividend; ſo that 7 is the Quotient without any Fraction, and the like is to be underſtood of dividing any other number by any other number: Or to give you an Example how to divide a larger Sum, and by conſequence any Sum; as to *Divide* 560472 by 968, by *Napiers Bones.*

Firſt, Upon Paper ſet down the Dividend, and making a ſtroke with your Pen on the left hand, there place the Diviſor,

and

and making another ſtroke with your Pen on the right hand, there place the Figures of the Quotient, then obſerve upon the Dividend how far the Diviſor will reach, as in this Example, to the Figure 4; there make a prick; Then having tabulated the Bones, placing the Diviſor in the Top as before, find out the neareſt leſs number to 5604. which you will find to be 4840. which you will find againſt the Figure 5, therefore put 5 in the Quotient, and place this 4840 under the Figures of the Dividend, and ſubſtract it from 5604, there remains 764, to which add the next Figure in the Dividend, *viz.* 7. (making a prick under the ſaid Figure in the Dividend, to know how far you are gone in the operation) Theſe together make 7647: Then upon the Bones look for the neareſt leſs number to 7647, and you will find it 6776, and over againſt it the Figure 7, therefore put 7 in the Quotient: And Subſtract this 6776 from 7647, the Remainder is 871, to which add the next Figure in the Dividend, *viz.* 2, and theſe make together 8712: Therefore on the Bones look the neareſt leſs number to 8712, but you will find the exact number it ſelf ſtanding againſt the Figure 9, therefore put 9 in the Quotient, and nothing remains, elſe what had remained, had been the Fraction, and this

will

will make plain any other operation in Diviſion upon *Napiers Bones*, and therefore I ſhall need to add no more.

968 (560472 (579
4840

7647
6776

8712

And thus much ſhall ſuffice to have ſpoken for the underſtanding of *Napiers Bones*. As alſo for what I ſhall ſay to the Firſt Part of the *Mathematicks*, conſiſting in *Arithmetick*, and the four ſeveral kinds of it, as it is diſtinguiſhed into *Common*, *Decimal*, *Logarithmetical*, and *Inſtrumental*; and here I intended to have ſhown the Uſe of the Reſt of the *Sector* (beſides the Lines of Numbers, Sines and Tangents): But upon more mature conſideration I find it more properly to be *Geometrical*, then *Arithmetical*, and therefore by way of Tranſition it will lead me by the hand (as it were) to the Second Part of the *Mathematicks*, (or the Second *Mathematical Liberal Science*) to wit, *Geometry*, where you may expect an Explanation of the reſt of the *Sector*, as alſo the way of *Protraction* by Scale and Compaſs.

The Second BOOK.

OF GEOMETRY.

PART I.

GEOMETRY is the Second *Liberal Science* which may be properly called *Mathematical*, and may be considered under a double Notion. First in General, as it makes and measures Geometrical Figures. Secondly according to the proper and particular Etymology of the Word, which is derived from γῆ *Terra*, & μέτρον *Mensura*, The Measuring of the Earth (that is as I have elsewhere explained it) the whole Globe of the Earth, containing Sea and Land, that which concerns the Land being called *Surveying*, and that which concerns the Sea, *Navigation*.

I. *But first of* Geometry *in General.*

All Bodies whatsoever which come within the Compass of *Geometry*, or are to be measured, may be distinguished into these Two kinds. First, *Superficies*. Secondly, *Solids*: And these are represented by *Geometrical* Figures, which Figures being protracted or laid down in due Proportion and Quantities answerable to the several Bodies they represent. The Bodies themselves are Measured by the same Rules as these Figures are which represent the same.

Now these Figures are as various, as there are Superficial or Solid Bodies of various and different shapes and magnitudes; insomuch as in this respect they may be said to be infinite, some Regular, but the most Irregular. All which, notwithstanding of what shape or Irregularity soever, may be reduced to some of these three Regular *Geometrical* Figures, the *Circle*, *Square*, and *Triangle*. Therefore in this place I shall shew you

1. How these three Figures are produced, protracted, or made.

2. How they may be reduced to one another, or any other to these or some of these. And,

3. How

3. How theſe, and conſequently all other Geometrical Bodies or Figures are to be meaſured, whereby you will ſee, that as *Arithmetick* is principally comprized in the *Rule of Three*, ſo *Geometry* and *Aſtronomy* may be comprehended in the Reſolution, or the making and meaſuring of a Triangle, called *Trigonometry*.

Firſt then, How theſe three Figures the Circle, the Square, and the Triangle are produced, and ſo they will be found to derive their Original from theſe three *Geometrical* Principles.

1. A point which is void of all magnitude, and hath neither length nor Breadth.

2. A Line which is a Length without Breadth, drawn from one point to another, and may be either ſtrait or crooked.

3. An Angle which is made by two Lines meeting in a point, that point being called the Angle, or the Angular Point: So that as the *Terms* or *Limits* of a Line are Points, and the *Terms* or *Limits* of Angles are Lines, ſo the *Terms* or *Limits* of Superficies are both Lines and Angles, and the *Terms* or Limits of Solids are Superficies.

Again, a Point hath neither Length nor Breadth; a Line hath Length, but no Breadth; a Superficies hath both Length and Breadth; a Solid hath both Length, Breadth, and Thickneſs.

CHAP. I.

BUT before I ſpeak particularly of the three *Geometrical Figures*, I ſhould neceſſarily lay down ſome few *Geometrical Problems* becauſe (as a Circle is made of a Line drawn equally diſtant on all ſides from a Point called its Center.) So the Square and the Triangle are made up of Lines and Angles, and therefore there will be ſeveral times mention made of Parallel Lines, and Perpendiculars: And alſo in the two latter Figures there will be occaſion for laying down of Angles, *&c.* I ſhall therefore in this place (once for all) lay down theſe few *Geometrical Problems*, not to trouble you with any more but thoſe abſolutely neceſſary.

SECT

SECT. I.

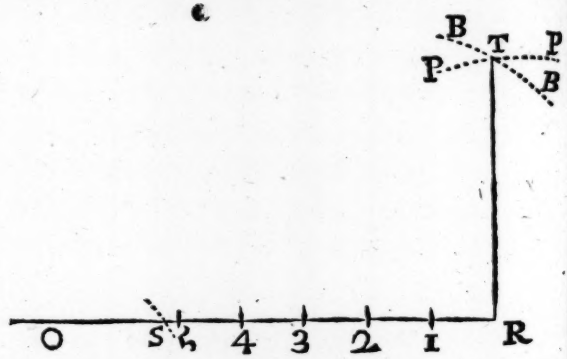
First, *To draw a Line Parallel to any other Line given at any distance required.*

Let the Line given be AB, unto which it is required to draw another Line Parallel thereunto at the distance AC, or BD. Open your Compasses to the distance AC or BD, and place one Foot in A with the other describe the Arch C, also place one Foot in B, and with the other describe the Arch D, then draw the Line CD, so that it may only touch the Arches C and D, so shall the Line CD be parallel to AB, as required.

SECT. 2.

To Erect a Perpendicular on the end of a Right Line given.

First, Divide the Line OR from R in any 5 Equal parts, as in the Example 1. 2. 3. 4. 5. Then take with your Compasses the distance from R to 4. and placing one Foot in R, with the other describe the pricked Arch PP, then take with your Compasses the distance from R to 5, and placing one Foot in 3, with the other describe the pricked Arch BB, and from the point where these two Arches intersect one another to the point R, draw a Right



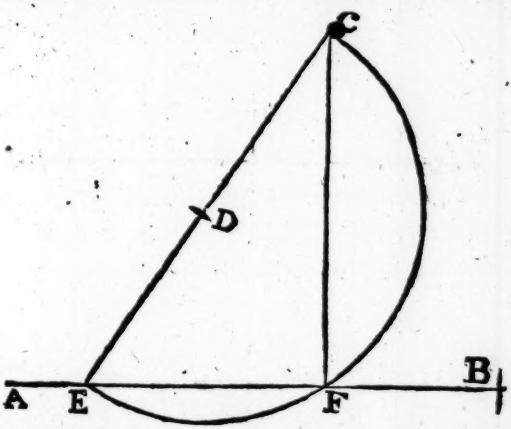
Line, and that will be a true Perpendicular; or open your Compasses to any convenient Distance, and set one Foot in the point

point R, with the other make a mark at pleaſure, as ⊙, then keeping the Compaſs point in ⊙, with the other deſcribe an Arch as BB, as alſo another Arch to cut the given Line in *S*. Laſtly lay a Ruler from *S* to ⊙, will croſs the Arch BB in T, and a line drawn from T to R will be the perpendicular required.

SECT. 3.

How to let fall a Perpendicular from any point aſſign'd upon a Right Line.

Let the Point given be C, and to let fall a Perpendicular upon the Line AB. Firſt from C draw a Line by chance, as CE, which divide into two equal parts at *D*,



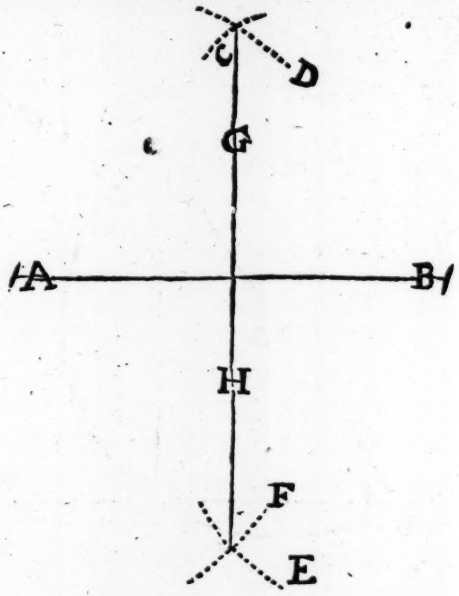
and with the Diſtance D C deſcribe the Semicircle CFE, and note where the ſame

cuts the Line AB, and from that point A, to the point C, draw the right Line C F, which ſhall be the Perpendicular required.

SECT. 4.

To Divide a Line into 2 equal parts by a Perpendicular.

Let the given Line be A B, fix one foot of your Compaſſes in A, and ſetting them at any convenient diſtance above half the given line, deſcribe the Arches D and F, carry your Compaſſes to B with the ſame



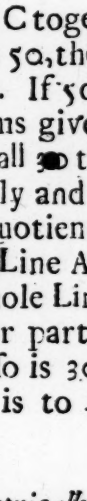
di-

diſtance deſcribe the Arches C and E, ſo as they may croſs the other Arches, and by thoſe Interſections lay your Ruler, and draw the line GH, which will cut the given Line in the midſt, and be a perpendicular alſo to it, which was required.

SECT. 5.

To Divide a Line given to any Proportion required, as to Divide the Line A *containing* 40 *Equal parts (let them be what they will) in proportion, as* 20 *is to* 30, *or as the Line* B *is to* C.

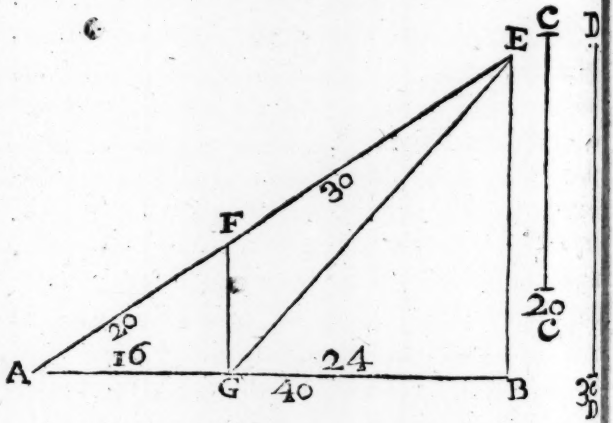
Add (the Lines B and C together) or 20 and 30, which makes 50, then ſay by the Rule of Proportion. If 50 the ſum of both the given Terms give 40 the whole Line A, what ſhall 30 the greater Term give? Multiply and Divide, you ſhall have in the Quotient 24 for the greater part of the Line A, which taken from 40 the whole Line, there remains 16 for the other part.



For as 50 is to 40, ſo is 30 to 24, or 20 to 16, or as 30 is to 20, ſo is 24 to 16.

But to do this *Geometrically*, or by Lines, ſuppoſe AB to contain 40, and you would divide the ſame in ſuch proportion as C to D.

First, From the point A, draw the Line A E at pleasure, making the Angle EAB, then take in your Compasses the Line C, and set it from A to F, also take the Line D, and set it from F to E, and draw the Line E B perpendicular to AB, Then from the point F draw the Line F G parallel to EB, cutting the given Line A B in G, so is the Line A B divided as required.



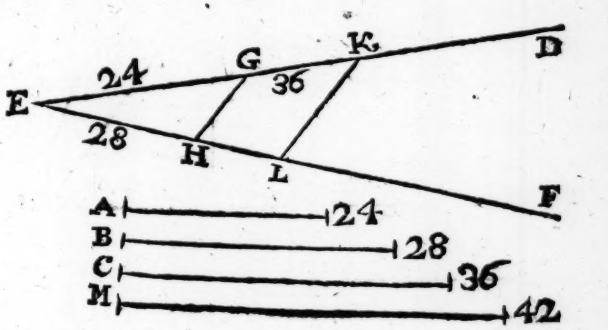
By dividing the Base of a Right Angled Triangle by this Problem, and drawing a Line from the opposite Angle to the point of Division at G, you divide the Triangle it self according to the same proportion.

SECT.

SECT. 6.

Three Lines being given, to find a fourth in proportion to them. Or to perform the Rule of Three *in Lines, Draw 2 Lines, making any Angle as the Lines* E D *and* E F, *then the Three Lines given, being* A B C, *to find the Line* M.

First take the Line A in your Compaſſes (that is 24 from a Scale of equal parts) and ſet it from E to G, then take the Line B 28. and ſet that length from E to H, then take the third given Line in your Compaſſes C. 36. and ſet it from E to K, and through the point K, draw the Line K. L parallell to G. H. ſo ſhall the Line E L be the fourth proportional required, For, as E G to E H, ſo is EK to EL.

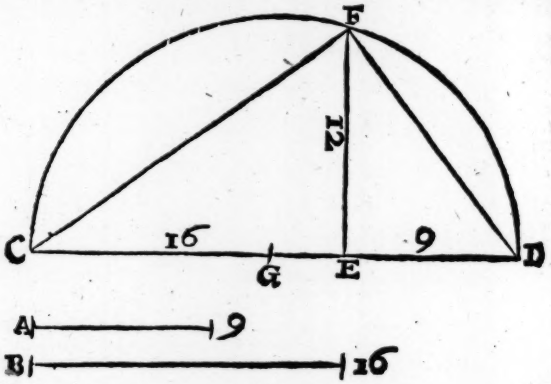


SECT.

SECT. 7.

To find a mean proportional Line between Two Lines given.

Let the two Lines given be A and B, which joyn together in the point E, making one right Line as CD. which divide into two equal parts in the point G, upon which point G with the distance GC or GD describe the Semicircle CFD, then from the point E, (where the two Lines are joyned together) raise the perpendicular EF. cutting the Periphery of the Semicircle in F, so shall the Line EF be a mean proportional between the two given Lines, A and B. For, As ED. 9. is to EF. 12. so is EF. 12. to CE. 16.



SECT. 8.

To Protract by Scale and Compaſs, or for want of a Scale to make a Line of equal Parts, which together with a Line of Chordes (the making whereof is taught Folio 67.) You may lay down any Line of what length you pleaſe: or any Angle of how many Degrees and Minutes ſoever, which take as follows.

Firſt, For laying down any Line of what length you pleaſe. There are ſeveral Scales both of Braſs and Wood, (that is) a Line of equal parts for protracting of Lines; and a Line of Chords for protracting of Angles; (together with other Lines at pleaſure, as a Line of Rumbs and Secants, or the like:) But for want of ſuch a Scale you may make theſe Lines your ſelf upon Paper, which will ſerve upon occaſion.

Firſt

First then, To make a Line of equal Parts, open your Compasses to some small distance at pleasure, and run it ten times over. Then opening the Compasses to the Extent of the whole Ten small Divisions run that distance over nine times more, so have you a Scale of equal parts, and one of them divided into other Ten parts. Then taking 6 of these larger Divisions in your Compasses, make it the Radius or Semidiameter of a Circle, whereby you may draw a Line of Chords, or Rumbs, or Secants, *&c.* as you'l see hereafter.

Now to Protract by these, you shall be taught it in making those two following Geometrical Figures, *viz.* Squares, and Triangles. To avoid unnecessary Repetitions, there are also *Protractors* (properly so called) by which you may lay down Angles, and avoid the drawing of many unnecessary Lines on the Paper; which said *Protractors* are made of a thin piece of Brass, cut into a perfect Semicircle, and divided into Degrees of a Circle, with a hole in the Center, and a Diameter drawn through the said Center, which is called the *Meridian Line.* The way of Protracting by this

Pro-

Protractor, you ſhall alſo have exemplified in making of Squares and Triangles. And in the third place you may alſo Protract Angles from a Line of Right Sines as ſhall alſo be ſhewed you hereafter; and ſhall give you examples of each of theſe ſeveral wayes in every Figure, ſo you may make uſe of which pleaſes you beſt. But now to proceed to ſpeak of thoſe three famous Geometrical Figures, The Circle, Square, and Triangle particularly, and in order, and that in a double Capacity. Firſt, as they are Superficies, And, Secondly, as they are Solids. And firſt of a *Circle*.

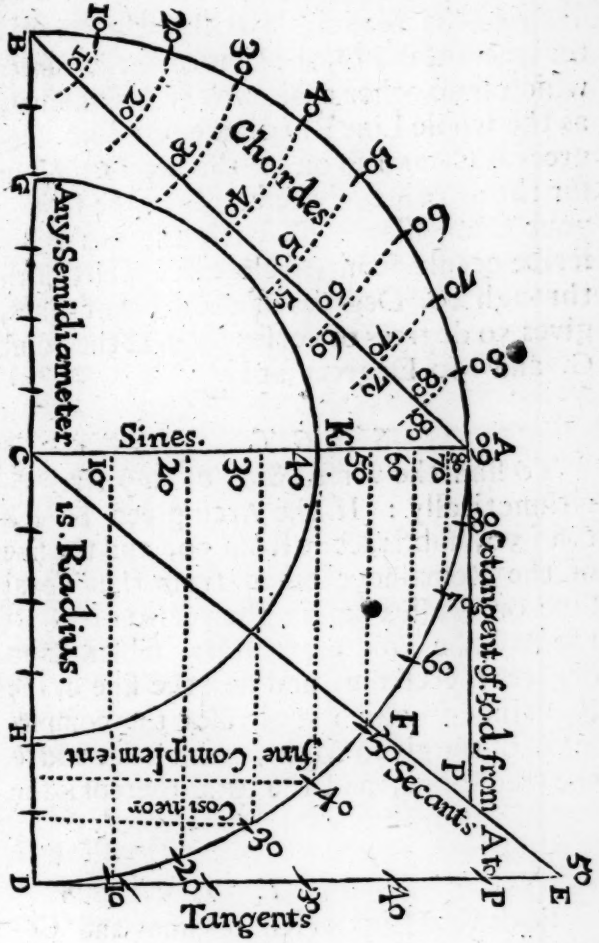
CHAP. II.

Of a Circle.

A *Circle* is a Geometrical Figure, which of all other is the moſt perfect, being an Emblem of Eternity, having neither Beginning nor End, and is made by the drawing a Line called the Circumference, equally every way diſtant from a Point in the middle called the Center, thorough which Point or Center a ſtraight line drawn from any part of the Circumference to another is called the Diameter; and any Line drawn

drawn from the ſaid Center to the Circumference is called the Semidiameter or Radius, or *Sinus Totus*, the whole Line of Sines, which being divided into 90 unequal parts are called Right Sines. The whole Circumference of the Circle, let it be bigger or leſs, is alwaies ſuppoſed to contain 360 Degrees, and is accordingly by Mathematicians alwaies divided into ſo many equal parts. The Semicircle into 180, and every Quadrant or fourth part of a Circle into 90. Each of theſe Degrees are ſubdivided into 60 Minutes, and each Minute ſuppoſed at leaſt to contain 60 Seconds, and every Second 60 Thirds, *&c.* as the largeneſs of the Circle will admit. Now by means of theſe Diviſions in the Circumference, for the finding out of Proportions, beſides the Line of Sines before-mentioned, there are many other Lines belonging to a Circle; as there are not only Right Sines, but alſo verſed Sines, Sines, and Sines complements, or Coſines; Tangents, and Cotangents, Secants, and Chords, for the underſtanding of all which, I ſhall give you this following Diagram, and how all theſe Lines are drawn and divided.

SECT.



SECT. 1.

First then, Setting one Foot of your Compasses in the Center at C, with the other describe

deſcribe the Semicircle BAD, the Diameter whereof is BD, the Line of verſed Sines; which is no other than two Scales of Sines, as the whole Line BD divided into 90. Degrees at C. and ſo on to 180. at D. Now for the drawing of theſe, place one foot of your Compaſſes at C, with the other deſcribe occult Semi-Circles: as GKH. drawn through 40. Degrees of the Line of Sines, gives 50 degrees of verſed Sines at the point G. and 130 Degrees at H.

SECT. 2.

To find the verſed Sine of any Arch Arithmetically: If the Arch given be leſs than 90, ſubſtract it from 90, and the ſine of the Remainder taken from the Total Sine or Radius leaves the verſed Line. If the Arch be greater than 90, ſubſtract 90 degrees therefrom, and ſeek the ſine of the Remainder, which is alwaies the complement of the given Arch; which ſine add to the whole ſine, and the total thereof is the verſed ſine of the given Arch deſired.

SECT. 2.

To draw a Line of Right Sines and Coſines through every Degree, or Tenth Degree of the Quadrant AD, draw right Lines parallel to the Line CD, till they cut the Lines AC, and thoſe are ſines: a Line drawn

per-

perpendicular from the ſame place to the line C D is the ſine complement or Coſine, as in the Diagram.

SECT. 4.

To draw a Line of Tangents and Cotangents. For Tangents upon the point D erect a perpendicular DE, and through every degree of the Quadrant AD draw right Lines from the Center C, till they cut the perpendicular DE, and theſe are Tangents. Another perpendicular erected upon the point A, and touching the aforeſaid line drawn from the Center, are Co-tangents, as AP.

SECT. 5.

To draw a Secant is thi every line drawn from the Center to E, which is the Secant of 50 degrees, the perpendicular it ſelf, which it cutteth, being the line of Tangents.

SECT. 6.

To draw a line of Chordes, Draw a ſtrait line from B to A, and ſetting one Foot of your Compaſſes in B through every Degree, or every tenth Degree in BA, draw Arches to cut the ſtrait line, and that ſhall be a line of Chords.

All theſe lines are of admirable uſe in finding of proportions, ariſing from that Symmetrical

metrical proportion there is betwixt a Circle, and the parts and lines belonging thereunto; for according to the length of the Radius or Semidiameter, ſo is the Circumference; and ſo are the Sines, Tangents, Secants and Chords belonging to the ſame Circle or Radius: For Sines and Tangents you ſee what excellent Effects they have joyned with the line of Numbers in *Inſtrumental Arithmetick*. And for the Line of Secants, it is that which conſtitutes the Meridian Line, or Line of Latitudes of abſolute uſe in Navigation; and for the line of Chords, its excellency conſiſts in protracting of Angles, without which (excepting the Circle, which is perfectly round,) no other Geometrical Figure can be made or meaſured.

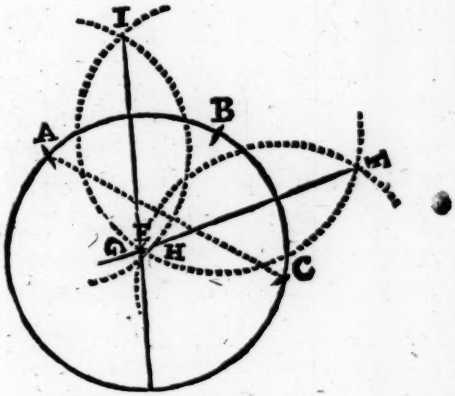
PROBL. I.

Three Points being given, not lying in a direct Line, to find the Center of a Circle, which ſhall cut all the 3 *Points.*

Let the Points be A B C, open your Compaſſes to ſome reaſonable Scantling more than half the diſtance betwixt A and B, with thatwideneſs ſtrike a Portion of an Arch upon the point A; do ſo alſo upon the point B, and note the interſections of

theſe

theſe 2 Arches, as at I and H, and produce a line from I by H infinitely. Then with your Compaſſes at the ſame diſtance as be-



fore, ſetting one foot in B; ſtrike an Arch towards C, and removing the ſame to C. ſtrike another Arch towards B, to croſs each the other, and where theſe interſect as at F and G, draw another ſtrait line to croſs the former; and where theſe two ſtrait lines interſect each other, as at E, is the Center of a Circle, which ſhall cut all the 3 points A.B.C, the thing required. The ſame Problem will alſo find the Center to any Segment of a Circle.

To

To Divide the Circumference of a Circle into any Number of equal parts. Divide 360 by the Number of Parts, the Quotient ſhews the Degrees as follows.

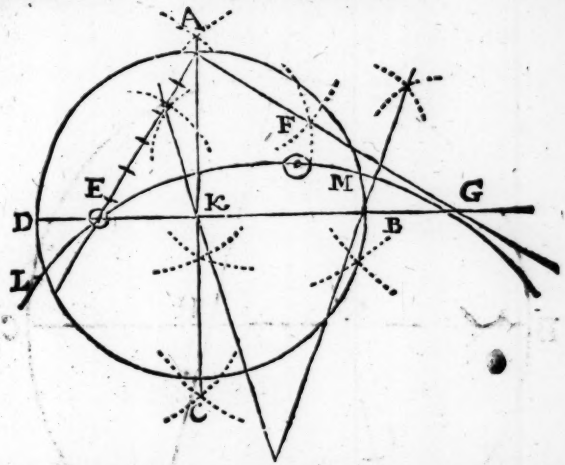
A Chord of	will divide the Circumference of a Circle into	Equal parts.
120		3
90		4
72		5
60		6
51 26 M.		7
45		8
40		9
36		10
32 44		11
30		12

And note that the Semidiameter will divide it into 6 parts, and the ſixth part of the Semidiameter into 36 equal parts, *&c.*

PROBL. 2.

Two Points within any Circle being given, how to deſcribe the Arch of another great Circle which ſhall paſs through thoſe two Points, and alſo divide the Circumference of the given Circle into two equal Parts.

Let the two Points be E, F, through either of which (as E) draw the right line DE, ſo as to paſs through the Center K: at right Angles draw the line AC, then draw the line EA, and from the point A erect a perpendicular AG, and ſo have you three points EFG, through which 3 points deſcribe

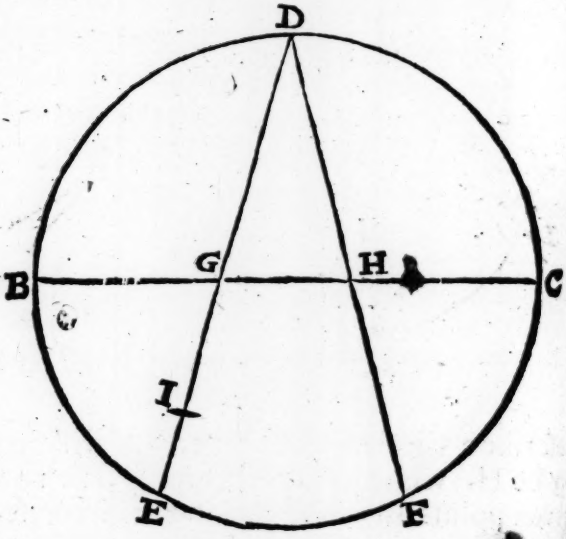


deſcribe a Circle whoſe Center you will find to be H, which will paſs through the two given points EF, and divide the circumference of the given Circle into 2 equal parts at L and M.

PROBL. 3.

To find a Right Line equal to the circumference of a Circle given.

Let the given Circle be BDCFE, Divide the upper Semicircle into halfes at D, and the lower Semicircle into three equal parts at F and E, and draw the lines DE, DF, which cut the Diameter of the Circle at G and H. Then make G I equal to GH. And the length D I is a very little more than the length of the quadrant BD, neither

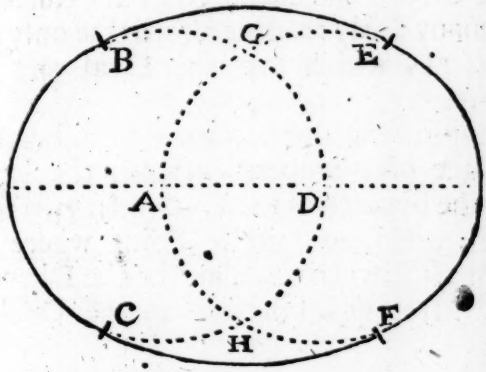


ther doth the excess amount unto one part of the Diameter BC, if it were divided into 5000, and four times the extent DI will be a little more than the whole circumference of the Circle.

PROBL. 4.

To draw a Geometrical Oval.

First, Draw two equal Circles, whose Centers are A and D, the Distance AD being equal to the Radius or Semidiameter of each Circle: Then open your Com



Compaſſes to the whole Diameter of the Circle, and ſet one Foot in H, with the other ſtrike the Arch B G E, and with the Foot of the Compaſſes removed to G ſtrike the Arch FHC, ſo will the Figure B C F E be a perfect Geometrical Oval.

CHAP. III.

HAving ſhewn you how a Circle is produced, with all the Parts and Lines belonging thereunto; I come to ſhew you in the next place, How other Figures may be reduced thereunto; and afterwards ſhall ſhew you how to meaſure a Circle either ſuperficial, or ſolid.

Now the Figures which may be reduced to a Circle, and meaſured by its Rules, are not many: My reading affords me only two material, which are the Oval and the Square.

And for the Oval divide the Diſtance on the line of Numbers between the length and the breadth of the Oval into two equal Parts, and the middle Point where the Compaſs ſtayeth on, ſhall be the Diameter of a Circle equal in Area to the Oval given.

SECT. 1.

To Reduce a Circle into a Geometrical Square, or by knowing the Superficial content of a Circle, to find the ſide of a Square equal to it.

Extract the Square Root of the whole content of the Circle, by taking half the Logarithme of the ſaid content. As ſuppoſe the content to be 616, the Logarithme whereof is 2,789581.

Take the half thereof, which is the Square Root 1.394790.

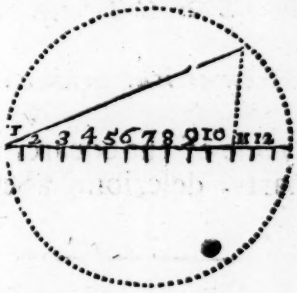
The nearſt number anſwering hereto is 25, ſo the ſide of the Square ought to be near 25 of the ſaid parts, whereof 616 were ſuppoſed to be the contents, whether Inches, Feet, Yards, *&c.*

SECT.

SECT. 2.

To Reduce a Circle into a Geometrical Square.

Having deſcribed a Circle, draw a Diameter, which divide into 14 parts, and on 11 of thoſe parts erect a perpendicular, which extend to the circumference, and



from the point of interſection, draw a line to the extream part of the Diameter, which line ſhall be the ſide of the Square, containing the proportion (or Area) of the Circle.

SECT. 3.

To Reduce a Square to a Circle Inſtrumentally.

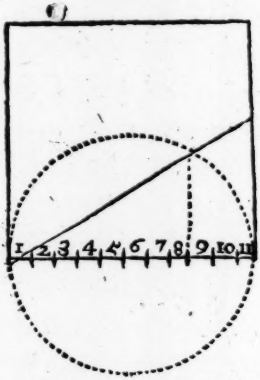
The ſide of the Square (as above) is the Square Root of a Number, which is the Superficial

perficial content of a Circle equal to the ſaid Square, and knowing the ſuperficial content, you may find the Diameter. If the Diameter be 10, the Superficial content upon the line of Numbers is 78.54 parts. Divide the diſtance betwixt the given content, and 78.54 into two equal parts. The ſame Diſtance the ſame way will reach from 10 to the Diameter required.

SECT. 4.

To Reduce a Geometrical Square into a Circle.

Take any ſide of the Square, and Divide it into 11 parts, deſcribing a Semicircle to



the ſaid Diameter: Then on 8 of thoſe parts erect a perpendicular, which extend

to

to the circumference of the Semicircle. Then draw a Line from the extream part of the Diameter, interſecting the perpendicular in the circumference of the Circle, and continue that line till it touch the ſide of the Square; ſo that line will be the Diameter of a Circle anſwerable to the Square given.

CHAP. IV.

TO conclude, (and to get out of this Circle) it remains only to ſhew you how to meaſure a Circle, and conſequently an Oval, a Square, or any other Figure reducible thereunto.

Firſt then for a Superficial or Flat Circle, ſuch as may be a piece of Ground, or Board, or Glaſs perfectly round, or any other Flat, Round Superficies. Take theſe Rules.

SECT. 1.

Having the Diameter, to find the Circumference.

Multiply the Diameter by 22, and Divide the Product by 7, as 28 by 22 is 616. Divided by 7 is 88.

SECT. 2.

Having the Circumference, to find the Diameter.

Multiply by 7, and divide by 22. As 88 multiplyed by 7, gives 616 divided by 22, is 28 the Diameter.

SECT. 3.

By the Diameter only to find the Area, or Superficial Content.

Multiply the Diameter by it self, as 28 by 28, that makes 784. This Product multiplyed the second time by 11, is 8624, which divide by 14, the Quotient will be 616, and that is the Area of the Circle.

For a half Circle, a Quadrant or any lesser portion of a Circle, whose Point goeth to the Center. Half the Arch multiplyed by the Semidiameter, produceth the superficial Content.

To measure the content of the Segment of a Circle, Take the chord $12\frac{2}{7}$, and the perpendicular 4, and multiply the whole of the one by two Thirds of the other, and it will come very near.

To find the Superficial content of a Circle, you need do no more but multiply half the circumference by half the Diameter. As the circumference being 44, the

half

half is 22, the Diameter 14, the half is 7, Multiply 22 by 7, and the exact content of the Circle is 154.

To find the whole Diameter of a Circle, by knowing part thereof, and the length of the chord crossing the Diameter in that part: Suppose part of the diameter to be 4, the chord intersecting it 12, Square one half of the chord $6\frac{1}{2}$ multiplyed by it self is 40, divided by 4 the part of the Diameter given, rests 10, which added to the said part, shews the whole Diameter to be 14, whether the Section be of a Circle, or of a Globe.

CHAP. V.

Of a Solid Circle, *such as are* Globes, Sphears, Bullets, Balls, *or the like* : *To find their* Solid Contents.

MUltiply the diameter of the Circle by it self, as 28 by 28, makes 784, this product multiplyed the second time by the diameter 28, makes 21952, which two Multiplications are called cubing of the diameter. Then multiply that Cube Number by 11, makes 241472, and divide this

laſt product by 21, produceth the *Solid Content* 11498 $\frac{4}{21}$.

SECT. 1.

To find the ſolid Content of a Globe another way.

Find firſt the Superficial content of a Globe, which to do, If you multiply half the diameter by half the circumference, the Product is the Superficial content of a Circle of equal diameter. As 28 multiplyed by 88 is 2464: And then if you Multiply the content of a Circle having the like diameter by 4, the Product is the Superficial content of a Globe, as 616 multiplyed by 4, produceth likewiſe 2464.

Now hereby to find the ſolid Content of a Globe.

Multiply the Superficial content by the ſixth part of the Diameter, the Product ſhall be the ſolid content of the Sphere.

SECT.

SECT. 2.

In like manner knowing the ſolid content of a Globe, to find the ſide of a Cube equal to the Globe.

Extract the Cube Root of the ſolid content of the Globe, which is done by taking the third part of the Logarithme of the ſaid ſolid content. And this Cube Root ſhall be the ſide of a Cube, which in ſolid contents ſhall be equal to the Globe given.

SECT. 3.

And to find the Superficial Content of the Segment of a Globe; ſay, If the whole Diameter 14 give the whole Superficial Content 616, what will 4 the part of the Diameter give, which you will find 176, and ſo of any other.

And thus much ſhall ſuffice to have ſpoken of a Circle, how it is produced, reduced, and meaſured.

Having hitherto treated of a Circle and Globe, I come now to ſhew you the Producing, Reducing, and Meaſuring of the other two principal Geometrical Figures, the *Square* and *Triangle*, as I have already done

concerning the Circle. And as the Circle was made up of a point, and a crooked or circular line drawn round about it; ſo *Squares* and *Triangles* are made up of *Lines* and *Angles*; *Squares*, and right lined *Triangles* of ſtreight *Lines* and *Angles*, *Sphærical Triangles* of crooked circular *Lines* and *Angles*: Now a ſtreight, or a right line is the neareſt diſtance drawn betwixt two points.

A Circular or ſphærical Line is part of the circumference of a *Circle* bigger or leſs.

An *Angle* in a *Sphærical Triangle*, is (as was ſaid before) the diſtance of two Lines meeting in the *Angular* point, and meaſured in the circumference of a *Circle* at the diſtance of the *Radius*, or *Semidiameter* of the ſaid *Circle* from the ſaid Angular point, which pray obſerve very well.

Now every *Circle*, whether great or little, doth contain 360 degrees, every degree containing 60 Minutes, ſo that a Semicircle contains 180 degrees, a Quadrant or fourth part of a *Circle* 90 degrees. Now if an *Angle* be a Quadrant of a *Circle*, and contains 90 degrees, it is called a *Right Angle*: If it contains leſs than 90 degrees, it is called an *Acute Angle*, if more than 90 degrees, it is called an *Obtuſe Angle*. Theſe *Angles* and *Lines* do conſtitute all kinds of Geometrical Figures whatſoever.

CHAP.

CHAP. VI.

Of a Square.

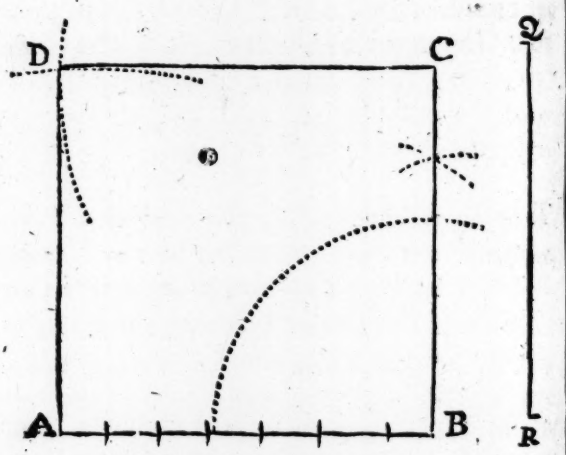
THere are ſeveral ſorts of four ſided Figures, as the *Long Square*, or *Parallelogram*, whoſe Angles are all equal, a *Rhombus*, and a *Rhomboides*, whoſe oppoſite ſides are all equal; and a *Trapezium* conſiſting of 4 unequal ſides, and 4 unequal Angles: But all theſe being reducible to 2 Triangles, or to a Geometrical Square, It is the Geometrical Square which I ſhall here ſpeak of, which conſiſts of 4 equal Streight Sides, and 4 equal Right Angles.

To make a Geometrical Square whoſe ſide ſhall be equal to a Line given, or to any Number of equal parts, ſuppoſing the ſame to be either Inches, Yards, Perches, or any other Meaſure.

As ſuppoſe you would make a Square, whoſe ſide ſhould contain 36 equal parts, (which ſuppoſe yards) or whoſe ſide ſhould be equal to the line Q R, which is the ſame. Firſt, From your line of equal parts, either upon your Scale, or prickt out upon Paper,

take

take with your Compaſſes three of the larger diviſions, which ſignifie 30, then extend them further to 6 more of the ſmaller diviſions, which together make 36, and prick out, and draw a line of the ſame length as A B: Then by the *Probl.* fol. 54, erect a perpendicular upon the end of the ſame line at B, containing the ſame length. Then keeping your Compaſſes at the diſtance of Q R, or A B, or B C, (all three being equal) ſet one Foot of your compaſſes in the point C, with the other



deſcribe the Arch at D: Alſo the compaſſes reſting at the ſame diſtance, place one foot in A, and with the other croſs the former prickt Arch in the point D. Laſtly,

where

where these Arches intersect one another, make a prick with the point of your compasses, and from thence draw right lines from D to C, and from D to A, which shall include the Geometrical Square, ABCD.

Now if instead of erecting the perpendicular BC (as is before directed) you protract an Angle of 90 degrees, or a right Angle at the poiut B, it will effect the same thing, and a line drawn at the same Angle will be a true perpendicular.

Now I told you before, there were 3 several ways of protracting an Angle.

1. By Scale and Compass, or a line of Chords.
2. By a Protractor properly so called.
3. By a line of Right Sines, and I shall shew how to protract an Angle of 90 degrees at the point B, every one of these waies.

SECT. 1.

First, By a line of Chords, and note that you must either have a line of Chords, whose Radius is shorter than the line AB, or DC, or else you must produce these lines till they meet with the Radius. Then with your Compasses set one foot at the beginning of your line of Chords, and the other extend

to

to 60 (where uſually are braſs Centers in moſt Scales) and with that extent ſetting one Foot in B, deſcribe an Arch of a Circle, cutting the lines AB and CB, and then from the points where this Arch interſects the lines AB CB, meaſure with your compaſſes in the ſame Arch the Chord of 90 degrees, which is the diſtance from the beginning of the line of chords to 90 degrees or the whole line, and making a prick there, lay your Ruler to the ſaid prick and the point B, drawing a line to the point B, and the Angle at B contained betwixt the two lines AB and B C ſhall be a Right Angle, or an Angle of 90 degrees, which was the thing required; and the like is to be done for any other Angle, of what quantity of degrees ſoever, as 30. 40. 50. 130. degrees, with their odd Minutes, if there be any; But I hope this Example may ſerve for all, or inſtead of many, ſo I ſhall mention no other in this place.

SECT. 2.

Secondly, By the *Protractor* do thus, put a Pin through the Center hole, fixing it in the point B, then lay the Meridian line of the Protractor upon the line A B, and by the edge of the Protractor make a mark againſt 90 degrees, and from that mark draw

draw a line to the point B, and the Angle included betwixt the two lines is an Angle of 90 degrees, as before. Suppoſe the Semi-circle BAD, (as in *pag.*65.) were a Protractor, the Limb thereof divided into degrees as there it is, the Point C. is the hole at the Center, the Line B D is the Meridian line, and at the Point A you find 90 degrees: The like is to be underſtood of any other Number of degrees whatſoever, more or leſs, or any other Protractor whatſoever. Lay the Meridian upon AB the line given, the Center upon B, and againſt 90 degrees, or the point A in the Protractor make a mark at C, and from C to B draw a Line, and AB, and CB will include an Angle of 90, degrees as required.

SECT. 3.

Thirdly, To Protract an Angle (which ſuppoſe 90 degrees) by the line of Right Sines, do thus. Firſt, Set one foot of your Compaſſes in the beginning of the line of Sines, and extend the other to the Sine of 30 degrees, with this extent ſet one foot of your Compaſſes in the Angular point at B, and with the other ſtrike an Arch of a Circle, cutting the line AB. produced to a ſufficient diſtance. Then taking 45 degrees from the beginning of the line of Sines in your

your Compaſſes, which is half of 90. (or the half of any other Angle required;) and ſetting one foot of your Compaſſes in the point of interſection where the Arch cutteth the Line AB, with the other foot of your Compaſſes make a mark in the ſaid Arch, and from that mark draw a right line to the Angular point B. ſo is the Angle included double to the diſtance upon the Arch, *viz.* 90 degrees. And what is here ſaid concerning the protracting of an Angle of 90 degrees, is in all Reſpects to be underſtood of any other Angle, or any Number of Degrees whatſoever, whether more or leſs, the ſame Rule of working being obſerved for one as well as the other.

And ſo much ſhall ſuffice to have ſpoken to the firſt particular concerning a Square, to wit, the Producing or Making of it.

CHAP.

CHAP. VII.

In the next place, I ſhall ſhew you the Reducing of it; or rather, the Reducing of other Figures to it, which take as follows.

SECT. 1.

To Reduce a Long Square to a Geometrical Square.

WHich is no more but to find a mean proportional betwixt the length and the breadth of the Long Square, for that ſhall be the length of a ſide of a Geometrical Square equal in content thereunto. As Suppoſe the length of the long Square to be 64, and the breadth 16, between which two the mean proportional is 32; ſo that a Geometrical Square, whoſe ſides are 32, will be equal to this long S[illegible]re.

Now to find a mean proportional b[illegible] *ommon Arithmetick*, or *Arithmetically*, is taught before by Logarithmes, (*page* 33.) By the Lines of Numbers, Sines, and Tangents, (*page* 43.) By Protraction, (*page* 60.) where you have the variety of doing

doing it all these several ways, to which I refer you.

SECT. 2.

To Reduce a Rhombus or Rhomboides to a Geometrical Square.

From one of the Obtuse Angles let fall a perpendicular, and a mean proportional between the perpendicular and the side whereon it falls is the side of a Square equal to the same.

SECT. 3.

To Reduce a Triangle to a Geometrical Square.

Is to find a mean proportional between half the Base and the Perpendicular, which shall be the side of a Square equal to the Triangle: As suppose the Base 72, half whereof is 36; and suppose the perpendicular be 25, the mean proportional will be 30. And so any other irregular Polygon, (or many-sided Figure) may be reduced into a Square, If first you Reduce the same into Triangles.

And thus much for Producing and Reducing a Geometrical Square: Now in the last place, To Measure it.

CHAP.

CHAP. VIII.

The laſt thing that remains to be ſpoke to about a Square, *is the Meaſuring of it; of which I come now to Entreat.*

SECT. 1.

To Meaſure the Square.

WHich is very eaſie and ſhort, Multiply one of the ſides in it ſelf, or by it ſelf, and the Product is the Superficial Content or Area: As 6 by 6 produceth 36, and 4 by 4 16, *&c.*

SECT. 2.

But beſides this Superficial, or Flat Square, *there is alſo a* Solid Square; *ſuch is a Cube, when the Top and the Bottom as well as the 4 ſides are all Squares in faſhion of a Dye.* Now to Meaſure this.

Firſt, Multiply the length by the breadth, that gives the Superficial Content, and that Superficial Content multiplyed again by the Depth, gives the Solid Content in Cube Inches.

CHAP.

CHAP. IX.

Of a Triangle.

HAving ſpoken to the two former Geometrical Figures, the *Circle* and the *Square*. In the third and laſt place I come now to ſpeak of a *Triangle*, which is the moſt admirable of all the Three; The Producing of which, the Reducing whereunto, and the Meaſuring whereof is the Sum and Subſtance of all the Mathematicks.

For hereby we meaſure both Sea and Land, all Regular, and all irregular Bodies; the very Heavens themſelves, and all the Heavenly Bodies: All Real and Imaginary Magnitudes, Heights and Diſtances, whether acceſſible, or inacceſſible, as in the following part of this Diſcourſe you may beſt diſcern, when we come to particular Application of a *Triangle*, to *Surveying*, *Navigation* and *Aſtronomy*.

Triangles are of Two Sorts: Either 1. Plain: Or, 2. Sphærical, each conſiſting of three Sides and three Angles, which together make ſix ſeveral Parts: Any three of which Parts being given, any, or all of the

the other three may be found out by the Golden Rule, or Rule of Proportion.

For Sphærical Triangles, they properly belong to Aſtronomy, and the Reſolution of Aſtronomical Queſtions; therefore I ſhall paſs them by in this place, and refer the Reader till I come to ſpeak of Aſtronomy, and ſhall then ſhew you the Reſolution of them; and how thoſe Queſtions about which they are converſant, may be reſolv'd Inſtrumentally, with far more eaſe, more pleaſure, and more plainneſs. At preſent I ſhall ſpeak only to Plain or Right Lined *Triangles*, which are ſufficient for *Surveying* and *Navigation*; for taking Heights and Diſtances both by Sea and Land.

Now of theſe, If any one of the Angles be a Right Angle, or 90 Degrees, It is called a Right Angled *Plain Triangle*. If any of the Angles be an *Obtuſe Angle*, or more than 90 Degrees, it is called an *Obtuſe Angled Plain Triangle*. If all the Angles be Acute leſs than 90 Degrees (which they muſt be if they be neither of the former,) then it is, called an *Acute Angled Plain Triangle*. And of theſe in my former Method as followeth.

CHAP.

CHAP. X.

1. IN the first place I shall therefo shew you, How to Produce, Pro tract, or Frame a Triangle.

2. I shall shew you how all other Figure whether Regular or Irregular may be Re duced to one Triangle, or however to ma ny, and so measured by its Rules.

3. How to Measure a Triangle, or t find the Contents either of a Superficial, o of a Solid Triangle. And likewise by fou of the six parts given, (*viz.* Sides and An gles.) To find the rest, which is properl called the Resolution of a Triangle, o Trigonometry.

First then, To Produce, Protract, La down, or Frame a Triangle, is no more bu the protracting or laying down of Line and Angles. The Lines or Sides are lai down from a Line of Lines, or equal Parts and the Angles are protracted by a Lin of Chords, or by a Line of Sines, or a Pro tractor, as you see is taught in the making of a Square, *pag.* 83, 84, 85, *&c.* But be cause there is only mention made of a Right Angle, (a Square admitting no other) I shal her

here give you Examples of other Angles alſo. Now you muſt know that every Acute-angled Triangle, whoſe Angles are all Acute, doth contain 180 degrees, all the Angles added together; A Right Angled Triangle, beſides the Right Angle, which is alwaies 90 degrees, contains other 90 in the two other Angles; the one whereof is alwaies the complement of the other to 90 degrees more, being in the whole 180 degrees, or a Semicircle exactly. An Obtuſe Angled Triangle, by how much the more the Obtuſe Angle is more than 90, by ſo much the other two are leſs than 90: All three together notwithſtanding making exactly 180 degrees as the former.

So that the Three Angles of every Triangle are equal to two Right Angles, or 180 degrees, whether it be an Acute, Obtuſe, or Right Angled Triangle.

As the Angles are thus diſtinguiſhed into Right, Obtuſe and Acute, ſo alſo are the Sides of a Right Angled Triangle (which is of moſt uſe and excellency) diſtinguiſhed into theſe three, The Baſe, the Perpendicular, and the Hypothenuſe. (In other Triangles, either Obtuſe, or Acute, they are called the Hypothenuſe, and the 2 Catheti.) Or others divide them into the ſubtending ſide, which is the ſide oppoſite to any Angle, and the two containing ſides

which

which are the ſides next on either ſide the ſaid Angle.

In a Right Angled Triangle, the Hypothenuſe is alwaies the ſide oppoſite to the Right Angle, the Baſe is uſually accounted the longer of the other two, and the perpendicular the ſhorter, including the Right Angle betwixt them.

Now to make a Triangle, whoſe Baſe is 400 inches, feet, or yards, and perpendicular 231, and Hypothenuſe 462; the Right Angle 90 degrees, the Angle at the Baſe 30 degrees, and conſequently the other Angle at the top of the perpendicular 60 degrees, being the complement of the other Angle at the Baſe to 90 degrees. Take this following Example,

Firſt, Draw a line at pleaſure A B, and with your Compaſſes take 4 of the larger equal parts from your Scale or Line of equal parts, and mark that diſtance upon the ſaid Line AB, ſignifying 400, then upon the Point B raiſe a Perpendicular or protract an Angle of 90 degrees (which is all one as is taught *page* 86. either by a Line of Chords, a Line of Sines, or Protractor. Then with your Compaſſes take 231 from the Line of equal Parts for the length of the perpendicular; and ſetting one Foot in the Angle at B, make a mark with the other for the length of the perpendicular: Then

upon

upon the Angle A, at the Baſe protract an Angle of 30 degrees on this manner.

SECT. 1.

Firſt, by a Line of Chords, ſet one foot of your Compaſſes at the beginning of the Line, and extend the other to 60 Degrees for the Radius: Then with this extent ſetting one foot in the point A, deſcribe the Arch DE, cutting the Baſe in E, and then ſetting one foot of your Compaſſes again in the beginning of the Line of Chords; extend the other to 30 degrees, (that being the number of degrees the Angle is to contain) and ſetting one foot of the Compaſſes in the point E, where the Arch does interſect the Baſe; with this extent of 30 degrees make a mark, then laying your Ruler to the ſaid mark, and the Angular point A, draw a line at length, or till it meet with the perpendicular, ſo ſhall the Angle at the Baſe contain 30 degrees, and the other at the Perpendicular 60, being the complement to 90 degrees.

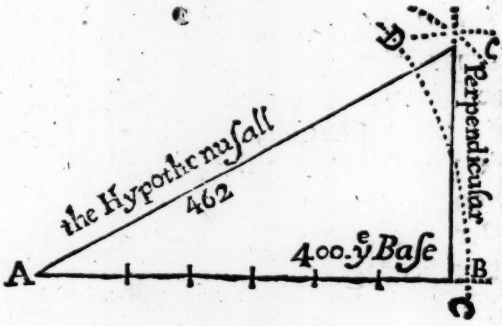
SECT. 2.

Now by the Line of Sines to protract the ſaid Angle of 30 degrees. Firſt, take in your Compaſſes 30 degrees of the line of

ſines, and at that diſtance draw the Arch, and at the diſtance of 15 degrees (which is half of 30 the Angle required,) draw the Hypothenuſal, ſo ſhall it be an Angle of 30 degrees.

SECT. 3.

And laſtly, To protract this Angle of 30 degrees by the Protractor, do thus: Prick the Pin through the Center hole in the point A, and lay the Meridian line upon the Baſe AB, and making a mark by the edge of the *Protractor* againſt 30 degrees, by the ſame mark, and the point A, draw the Hypothenuſal AC.

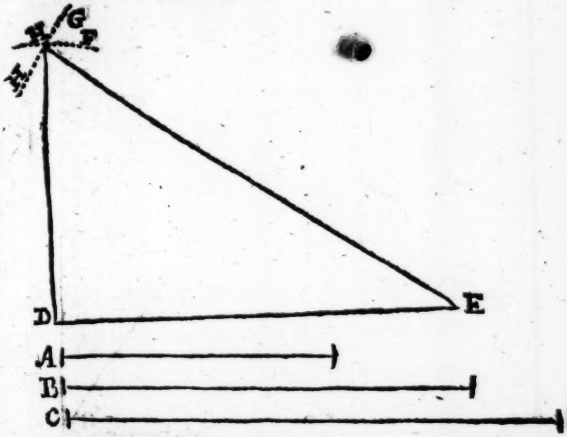


So that by any of theſe three ways you may protract an Angle, and any other Angle as well as this in the Example; obſerving the ſame Rules, which together with the

the ſides protracted from a line of equal parts, makes up the compleat Triangle, ABC; and by conſequence any other Triangle whatſoever: Only in an obtuſe Angle, where it is more than 90 degrees, as ſuppoſe 140, you muſt firſt prick out 90 degrees, and from thence the remainder that is 50 degrees more, which together make 140: And the ſame Rule is to be obſerved for odd Minutes, as is here given for degrees all along.

SECT. 4.

Three Lines being given, ſo that the two ſhorteſt together be longer than the Third, to make thereof a Triangle.



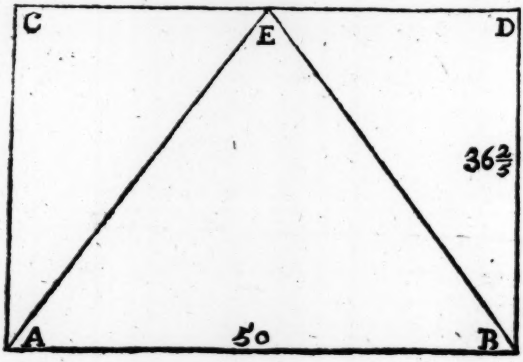
As ſuppoſe the Lines given be A, B, C, Firſt draw the line DF equal to the line B, then take with your Compaſſes the line C,

and ſetting one foot in E, with the other deſcribe the Arch HG: Alſo taking the given line A in your Compaſſes, and placing one foot in D, with the other deſcribe the Arch HF, cutting the former Arch H G in the point H. Laſtly, if from the point H you draw the lines H E and H D, you ſhall conſtitute the Triangle HDE, whoſe ſides ſhall be equal to the three given lines, ABC.

SECT. 5.

To make a Triangle to contain 910 *equal parts, (ſuppoſe Perches) whoſe Baſe ſhall be* 50 *of the ſame equal parts, be they what they will.*

Double the number of Perches or equal Parts 910, and they make 1820; divide this



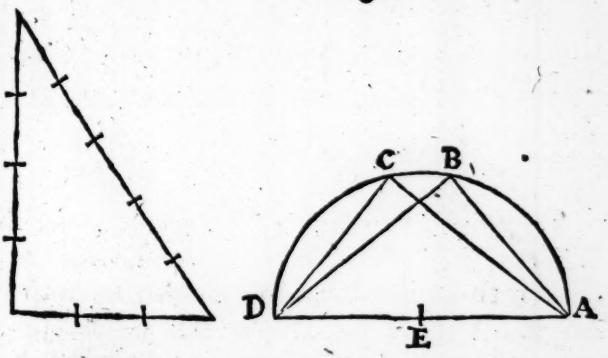
by the Baſe 50, the Quotient will be the length of the perpendicular of the Triangle,

gle, to wit, $36\frac{2}{5}$. Then from any Scale of equal parts lay down AB the Baſe 50, then upon B raiſe the Perpendicular BD. $36\frac{2}{5}$; and draw the line CD parallel to AB: Then from any point in the line CD (as from E) draw the lines EA and EB, including the Triangle EAB, which ſhall contain 910 equal parts as required. And the like, if you would make a Triangle to contain any other number of Perches or equal Parts, the Rule is the ſame *mutatis mutandis*.

SECT. 6.

To make a Right Angle readily.

Take a Line for the Hypothenuſal, and divide it into five equal parts, then take



three of thoſe equal parts for the Baſe, and four for the Perpendicular, and joyn theſe

together: Or, draw a Semicircle, as ABCD upon the Center E, (D A the Diameter) and from A draw a line to the Circumference, as at B or C: Then from B or C draw lines a to D, ſo have you made a right Angle, as ABD, or ACD.

SECT. 7.

To Divide a Triangle into any proportion whatſoever.

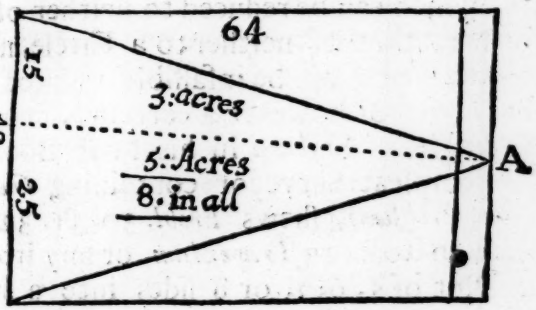
By the Fifth *Geometrical Problem*, (*p.* 58.) Divide the Baſe of the ſaid Triangle in proportion, as 20 is to 30, as there it is done, and from the point of Diviſion at G draw a line to the oppoſite Angle at E, ſo is the Triangle it ſelf divided in the ſame proportion; for as the line C 20 is to the line D 30, ſo is the Triangle EAG to the Triangle EBG.

To do this Arithmetically.

Suppoſe the whole Triangle to contain 8 Acres, and if it be required to Divide the ſame into 2 parts by a line drawn from the Angle A, the one part to contain 5 Acres, the other three. Firſt meaſure the whole length of the Baſe, which ſuppoſe 40, then ſay by the Rule of Proportion. If eight Acres

(the

(the Quantity of the whole Triangle) give 40 (the whole Baſe) what parts of the



Baſe ſhall five Acres give? Multiply and Divide, the Quotient will be 25 for the greater Segment, which being deducted from 40 (the whole Baſe) there will remain 15 for the leſſer Segment, then draw a line from the oppoſite Angle, which ſhall Divide the Triangle according to the proportion required.

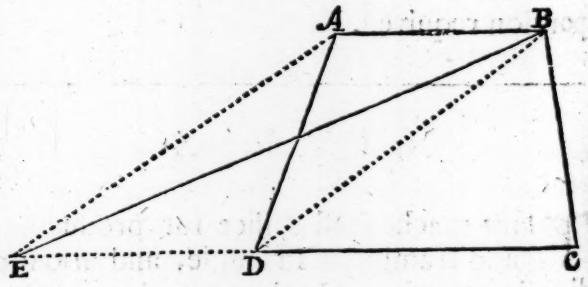
CHAP. XI.

THus much ſhall ſuffice for producing, and framing a Triangle, and alſo for dividing the ſame: In the next place according to my former Method, to ſhew how other Figures may be Reduced to Trian-

gles: And herein appears the excellency of a Triangle above the reſt, that all Geometrical Figures, whether Regular or Irregular, which can be reduced to neither of the former, that is, neither to a Circle, nor a Square; may yet be infallibly reduced to a Triangle, or however a certain Number of Triangles. *Leyburn* in his Firſt Book of his compleat Surveyor containing *Geometrical Problems*, ſhews *Probl.* 30. 31. 32. 33. How to Reduce a *Trapezium*, or any irregular Plot of 5, 6, 7, or 8 ſides into a ſingle Triangle, which I abridge in this manner.

To Reduce a Trapezium into a Triangle containing 4 unequal ſides.

Extend the line DC, and draw the Diagonal B. *D* Then from the Point A draw

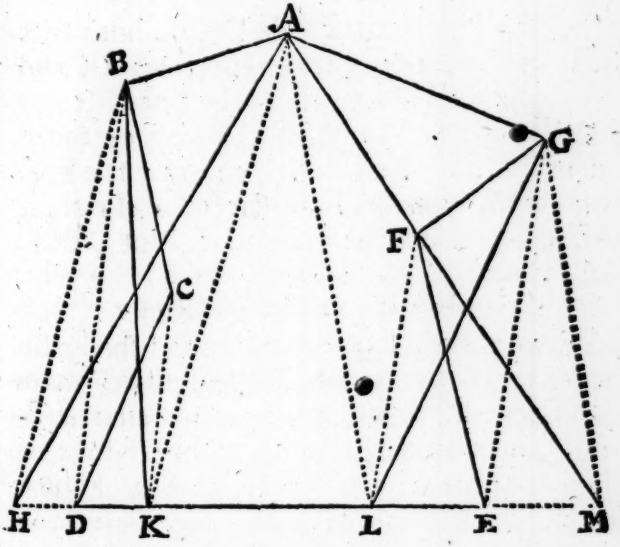


the line AE parallel to BD, extending it till it cut the ſide CD in the point E. Laſtly

From

From B draw the line BE, conſtituting the Triangle EBC, which ſhall be equal to the Trapezium, ABCD.

How to Reduce an Irregular Plot of 5. 6. 7. *Sides into a Triangle.*



Let ABCDEFG. Firſt Reduce this Figure of 7 ſides into a Figure of 5 ſides, keeping the ſame quantity, ABKLG on this manner. Draw the line BD, and parallel thereunto CK, then draw the line BK, whereunto the two lines BC and CD are thereby reduced. Then draw the line GE, and parallel thereunto FL, then if you draw the line GL,

the two ſides, GF and FE ſhall thereby be reduced to one ſtraight line, *viz.* GL, and the whole Plot to a Figure of five ſides ABKLG.

Now to Reduce this to a Triangle.

Firſt, Produce the ſide DE on both ſides at pleaſure, then draw the lines AK and AL, and parallel to them the lines BH and GM, cutting the line DE, being extended in H and M. Laſtly, If you draw the lines AH, AM, you ſhall conſtitute a Triangle AHM equal to the Irregular Plot ABCD EFG.

And having ſhewn you to Reduce theſe Figures into a ſingle Triangle, I ſhall alſo ſhew you how to Reduce the like Figures into ſeveral Triangles, which being ſeverally meaſured and added together, give the content of the whole Plot or Figure.

Now for the doing hereof obſerve this general Rule, That the Number of Triangles contained in any Figure or Plot, will be alwaies two fewer, or not ſo many by Two as the number of Angles, or Sides which the ſaid Figure or Plot doth contain.

As any Square or Trapezium which contains 4 Angles, and 4 Sides may be eaſily reduced to 2 Triangles drawn from Corner to Corner, ſo likewiſe a Plot of 5. 6. 7. or

8 ſides

8 ſides may be eaſily reduced to 3. 4. 5. or 6 Triangles, as will appear upon practice, and needs no *Example*. Now theſe Triangles meaſured particularly, and then the whole added together, gives the content of any Irregular Plot or Figure whatſoever.

CHAP. XII.

THerefore in the third place to ſhew you How to meaſure a Triangle, and by conſequence any other Plot or Figure, take this Rule, that half the perpendicular and the whole Baſe, or half the Baſe, and the whole perpendicular multiplyed the one by the other, gives the Area or Superficial Content of the ſaid Triangle.

SECT. 1.

Now the perpendicular of a Triangle is a Plumb line let fall from any Angle upon its oppoſite ſide, which we call the Baſe: How to let fall this perpendicular from any Angle or Point you have *Probl.* 3. *pag.* 55. to which I refer you.

Now for an Equilateral Triangle, or a Triangle having three equal ſides, obſerve this

this Rule: That the power of the ſide is to the power of the perpendicular let fall from any of the Angles to the ſubtendent ſide in proportion ſeſquitertia, or as 4 to 3, or $\frac{3}{4}$ of the other. Note the Power of a Line is the Square thereof.

SECT. 2.

Any three ſides of any Triangle being knowe, to find the Perpendicular: The greateſt ſide being aſſigned for the Baſe, upon which the perpendicular ſhall be ſuppoſed to fall: Firſt find the Sum of the other two ſides, (which ſuppoſe 13 and 11.) Then obſerve the difference betwixt theſe two ſides, which is 2; that done, ſuppoſe the whole Baſe to be 20: Say by the Rule of Proportion, As the whole Baſe 20 is to the Sum of the other two ſides added together, *viz.* 24, ſo is the difference of the other two ſides, *viz.* 2. to a fourth number $2\frac{2}{5}$, which being deducted out of the Baſe, the perpendicular will fall in the middle of that which remains, $2\frac{4}{10}$ deducted from 20, reſts $17\frac{6}{10}$, the half whereof is $8\frac{8}{10}$: a line drawn from that point from the Baſe to the oppoſite Angle is the true perpendicular to the ſaid Triangle.

Ano-

Another way to find the Perpendicular of an Equilateral Triangle.

Multiply one of the ſides by 13, and divide the Product by 15, the Quotient is the perpendicular, ſaies *Brown* in his Triangular Quadrant.

CHAP. XIII.

THus much for meaſuring a Superficial Triangle, now a ſolid Triangle is that Figure, or Solid Regular Platonick body, called a *Tetrahedron*, contained under 4 equal and Equilateral Triangles, or a Triangular Pyramid: The ſolid content whereof is found by Multiplying the Area or Superficial content of the Baſe by one third part of the length of the Pyramid or Tetrahedron from the midſt of one plain to the Apex or Top of the Solid Angle oppoſite thereunto.

Note that this Triangular Pyramid is little more than $\frac{1}{7}$ of a Cube of equal Baſe and Altitude: A Cylinder is $\frac{11}{14}$ of a Cube of equal Baſe and Altitude.

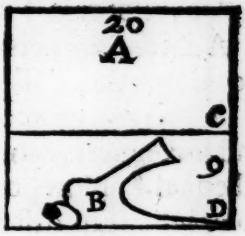
A Globe is $\frac{11}{21}$ of a Cube, or $\frac{2}{3}$ of a Cylinder, whoſe ſides and diameters are equal.

How to Meaſure the Solid Content of any Body, how irregular ſoever it be, The Form and Faſhion not regarded.

To perform this, you muſt prepare an hollow Cube, into which put your Irregular Body, which being placed therein, you ſhall pour in ſo much Water, till it no more than juſt cover the Body in the Cube, then make a mark in the inſide of the Cube where the Superficies of the Water toucheth. This done, take out the irregular Body, and mark again directly under the former, where the Brim of the Water now touches; for the diſtance of theſe two marks multiplyed by the Square of the Cubes ſide, produceth the Craſſitude of that Irregular Body. For Example.

Suppoſe

Suppoſe A to be the Cubical Hollow Veſſel, whoſe inward ſide ſuppoſe to be 20 inches, B the irregular Body, whoſe Solid Content is required. Firſt then I put B into the Hollow Cube A, and pouring in water till it be covered, admit the top of the water to reach to C: Then taking out the irregular Body again, admit the Superficies of the Water fall to D, then meaſure the diſtance betwixt C and D, ſuppoſe 9 inches, which Multiplyed into 400, the Square of the Cubes ſide produceth 3600; and ſo many Cubical Inches are contained in the Irregular Body B. By this you may meaſure any Part or Portion of a Solid Body.



20
20
400
9
3600 Inches.

Now having ſpoken of a Solid Circle, *viz.* a Globe, and of a Solid Square, *viz.* a Cube: And now of a Solid Triangle, *viz.* a *Tetrahedron.* Theſe two laſt being two of the five Platonick Bodies, I ſhall add the other three.

An *Octaedron* is contained under eight equal and equilateral Triangles.

An

An *Icosaedron* is contained under 20 equal and equilateral Triangles.

A *Dodecaedron* is contained under 12 equal Equilateral and Equiangular *Pentagons.*

A *Polygon* is any Figure of many sides, and a Regular *Polygon* is one whose sides are all equal. Now to measure any Regular *Polygon.* The distance from the Center to the middle of one side, and the half Sum of the measure of all the sides multiplyed together, shall be the true Area or Content thereof. To measure any Irregular *Polygon,* see *pag.* 105, by Reducing it to a Triangle.

If the Area or Content be 100, *then the sides of these Regular Figures are as follows.*

An Equilateral Triangle,	its perpendicular	13	123
	its side	15	2
A Square its side.		10	
A Pentagon, which is a Figure of 5 sides.		7	62
An Hexagon of 6 sides, its side		6	2
An Heptagon of 7 sides, its side.		5	26
An Octagon of 8 sides, its side.		4	55
A Nonagon of 9 sides.		4	3
A Decagon of 10 sides. 3 6		3	64
The Semidiameter or Radius of a Circle.		5	64

A Cone differs from a Pyramid, because a Cone hath alwaies a round Base and Superficies like a Sugar-loaf: But a Pyramid hath an Angular Base and Superficies of several

veral ſides: The way to meaſure both as to the Solid Content. Firſt, find the Superficial content of the Baſe, and Multiply the ſame by the third part of a perpendicular from the ſharp end to the Baſe. And according to theſe Proportions beforementioned is any other Quantity or content to be found out: As ſuppoſe I would have a Triangle to contain 200, what muſt the ſides be? The half diſtance in the line of numbers betwixt 100 and 200 will reach from 15.2 to 21.5 for its ſide, and from 13.123 the perpendicular of a Triangle, whoſe Area is 100; the ſame extent will reach to 18.6 the perpendicular of an Equilateral Triangle, whoſe Area is 200, *&c.*

SECT. 3.

The three ſides of a right-lin'd Triangle being given, to find the Superficial Content.

Add the three ſides together, (as 20. 13. 11.) which make 44, the half of which is 22; from which half Sum ſubſtract each ſide ſeverally to the end you may have the difference betwixt that half Sum and each ſide, which will be 2. 9. 11. Then add the Logarithmes of the ſaid half Sum and of thoſe differences together, and laſtly, Divide the Sum of all thoſe Logarithmes

garithmes by 2, so you will have the Logarithme of the Superficial Content or Area of the Triangle.

The half Sum	22.	1.342423
The Differences	2.	0.301030
	9.	0.954243
	11.	1.041393
The Sum of the Logarithmes		— .3.639089
The Area or Content req.	66.	1.819544

OF

OF GAUGING ALL SORTS OF VESSELS.

GAuging of Veſſels is no more but Meaſuring of Solids. Theſe Solids are to be Reduced to ſome Regular Geometrical Figure (as is taught before when we treated of Meaſuring of Solids in General) eſpecially to one of theſe Three.

1. The Solid Square or Cube.
2. The Solid Triangle, or any Irregular Solid.
3. The Solid Circle or Cilender.

1. *To*

1. *To Meaſure a Solid Square, or any Veſſel in the form of a Cube.*

The Rule is this.

Multiply the Length by the Breadth, and that Product by the Depth (in Inches) and that gives the contents in Square Inches.

Now to bring theſe Inches into Ale, Wine, or Corn Gallons.

Obſerve this Rule once for all, That formerly the Ale Gallon was accounted to contain 288¼ Cube Inches. But by the care and pains of Mr. *Nicholas Gunton*, the juſt quantity of the Quart for Ale remaining in the hands of the Chamberlains of His Majeſties Exchequer appears to be 70½ Square Inches, which makes the Gallon 282, as ſome have of late found to their no ſmall coſt. And I am of opinion (ſaith *Mayne*) that if the Wine Gallon were carefully examined, (and ſo alſo the Corn Gallon) they would prove to contain leſs than they are commonly holden to do.

But to proceed, for Ale divide the Product in Square Inches by 282; for Wine by 231, and for Corn by 272¼, and the Quotient of each is the Gallons; and if any

any Fractions remain to Reduce it into Pints, multiply the Numerator by 8, (ſuppoſe $\frac{128}{282}$ be the Fraction) that is 128 by 8, and Divide the Product by the old Denominator 282, the Quotient ſhews the Pints. Thus much for meaſuring the Square or Cubical Veſſel, you may alſo find *pag.* 89, 90. what other figures may be reduced to this, as a Long Square, a Rhombus, or Rhomboides, a Triangle, a Cirele, with its Segments, *&c.*

2. *Now in the ſecond place to Meaſure a Solid Triangle, or any Veſſel of a Triangular Form.*

The Rule is,

Multiply the perpendicular by half the Baſe, or the whole Baſe by half the perpendicular, and divide the Product by 282, the Quotient will be the Ale Gallons contained in one Inch of Depth upon that Triangle. If the Veſſel be of an Irregular Form, Divide it into ſeveral Triangles, and let fall perpendiculars in each, and ſo find their ſeveral Areas and Contents, and then add them together: So have you the Content or Area of the whole Figure, be it never ſo Irregular.

3. *In*

3. *In the third place to measure any Vessel which is of a Circular or Cylindrical Form; That is to say, Either Cylinders, Or, To be Reduced to Cylinders.*

The Rule is this,

Multiply half the Diameter by half the Circumference, gives the Area or Superficial Content of the Base or Bottoms, and then Multiply that Product by the Depth, gives the Solid Content in Inches, which Square Inches you may bring into Gallons, and Pints, as before is taught.

To find the Diameter of any Circle by the Circumference.

Multiply the Circumference by 7, and Divide the Product by 22.

To find the Circumference by the Diameter.

Multiply the Diameter by 22, and Divide the Product by 7.

To find the Area or Superficial Content of a Circle by the Diameter.

Square the Diameter, that is, Multiply it by it self, then Multiply the Product by

by 11, and divide by 14, the Quotient gives the Area or Superficial Content: For as 14 is to 11, ſo is the Square of the Diameter to the Area, or at leaſt the neareſt Approximation to it for Common Practice, or which comes infinitely near the Truth. As Unity is to 3.14159, ſo is the Square of the Semidiameter to the Area in Square Inches. Or,

Take this ſhort Rule to find the Ale or Wine Gallons in any Circle by the Diameter, and one Inch in depth.

Square the Diameter, (that is, Multiply it by it ſelf,) and divide that Square or Product by 359 for Ale Gallons, or by 294 for Wine Gallons; and then Multiply that Area or Superficial Content by the Length or Depth of the Veſſel, and that will give you the Solid Content of the whole Veſſel in Gallons.

Thus far we have ſuppoſed the Veſſel to be meaſured to be perfectly Circular or Cylindrical; and the Top and the Bottom to be both of the ſame Diameter: But if the Top and the Bottom be of different Diameters, (or which is the ſame) the Head and the Bung, then you are to find a Mean Diameter,

Which when the Veſſel is a perfect Cone, and the ſides ſtrait.

Firſt, Find the Area of the Baſe or Bottom, and multiply that Area by the third part of the Altitude, the Product is the ſolidity of the whole Cone.

If the Veſſel be not a whole Cone, but only a Part or Fruſtum of a Cone.

Add the Diameters at Top and Bottom together, and take the half of that Sum for the mean Diameter.

To Meaſure a Veſſel, which is an Irregular Cylinder, (ſuch as a But, Pipe, Hogshead, Barrel, &c.) find the Mean Diameter by theſe Rules.

1. Add double the Bung Diameter to once the Head Diameter, and Divide their Sum by 3, the Quotient take for your mean Diameter.

2. More exactly, To the doubled Square of the Bung Diameter, add the Square of the head Diameter, and that Sum Multiply by the Cask length, the laſt Product Divide by 1077, the Quotient is the Ale Gallons; or by 882 Quotes the Wine Gallons contained in that Cask.

3. *By*

3. *By Mr.* Oughtred's *way of Meaſuring the Fruſtum of a Spheroid.*

Firſt, take the Diameter at the Bung, and find the Area of the Circle anſwerable thereunto, and take two thirds of that Area. Secondly, Take the Diameter at the Head, and find the Area of that Circle, and take one third of that Area; add theſe two Sums together, and multiply the whole by the length of the Veſſel, the Product gives the content of the whole Veſſel in Cubique Inches, which divided by 282 for Ale, or 231 for Wine, gives the Gallons.

To Meaſure a Veſſel, which is an Irregular Cylinder, ſuch as a Butt, Pipe, Hogshead, Barrel, or the like.

Take the Diameter at the Bung, which ſuppoſe 23 Inches, and then take the Diameter at the Head, which ſuppoſe to be 20, the difference is 3: Now (in reſpect to the bending of the Staves in proportion, as 7 to 10) Multiply this difference 3 by 7 makes 21, and divide the Product by 10, makes $2\frac{1}{10}$, which added to the leſſer Diameter 20, makes $22\frac{1}{10}$ for the Mean Diameter: Half of this Mean Diameter multiplyed by half of its proportional Circumference,

ference, gives the Area or Superficial Content, and that multiplyed by the Length of the Veſſel, gives its Solid Content in Inches, and thoſe Inches reduced to Gallons by dividing by 282 for Ale, and 231 for Wine, *prout ſupra.*

To find a Mean Diameter by Gunters *Line of Numbers.*

Extend the Compaſſes from the Gauge Point to the mean diameter, the ſame Extent (twice repeated) will reach from the Length of the Cask, to the whole Content of the Veſſel. This Gauge Point is at 17.2 for Ale, almoſt 19 for Wine.

Mr. *Mayne* (in his little Book, called his *Practical Gauger*, which I recommend as the plaineſt, and eaſieſt, and ſhorteſt I ever met with,) beſides very good Rules for Gauging all ſorts of Veſſels, and the uſe of the Gauging Rule, which is very well done. He hath (beſides theſe) 4 very uſeful Tables, which I ſhall here mention, but refer the Reader to the Book it ſelf.

1. The Firſt is a Table of Areas of Circles in Ale Gallons and Milleſimal Parts, to every Quarter of an Inch, from 10 to 144 Inches Diameter. This is to find the Contents of the Fruſtum of a Spheroid in Ale Gallons, *pag.* 29.

2. A Table

A Table of one Thirds of Area's of Circles in Wine Gallons, calculated to every Quarter of an Inch, from 10 to 60 Inches Diameter, *pag.* 39. This Table is to find readily the Contents of a Wine-Hogſhead, or Beer-Barrel in Wine Gallons.

3. A Table of the Contents of Cylinders in Ale Gallons, and Centeſimal Parts from 12 to 60 Inches Diameter, and to 8 Inches in Depth ready caſt up, ſo that you have nothing to do but to find the Plain Diameter, or the Mean Diameter of any Cylindrical Veſſel.

4. In the Fourth place, A Table of Area's of Segments of a Circle: The Radius divided into 100 parts, calculated to the $\frac{1}{10000}$ part of a Square Inch. Hereby you may find the Vacuity, Ullage, or Wants in a Cask partly full, lying with his Axis parallel to the Horizon: The Cask being taken as the Fruſtum of a Spheroid: The Table and uſe of it follows.

A TABLE of Area's of Segments.

V.	Area	V.	Area	V.	Area	V.	Area
1	.0017	26	2066	51	5127	76	8155
2	.0048	27	2178	52	5255	77	8263
3	.0087	28	2292	53	5382	78	8369
4	.0134	29	2407	54	5509	79	8473
5	.0187	30	2523	55	5636	80	8576
6	.0245	31	2640	56	5762	81	8677
7	.0308	32	2759	57	5888	82	8776
8	.0375	33	2878	58	6014	83	8873
9	.0446	34	2998	59	6140	84	8967
10	.0520	35	3119	60	6265	85	9059
11	.0598	36	3241	61	6389	86	9149
12	.0680	37	3364	62	6513	87	9236
13	.0764	38	3487	63	6636	88	9320
14	.0851	39	3611	64	6759	89	9402
15	.0941	40	3735	65	6881	90	9480
16	.1033	41	3860	66	7002	91	9554
17	.1127	42	3986	67	7122	92	9625
18	.1224	43	4112	68	7241	93	9692
19	.1323	44	4238	69	7360	94	9755
20	.1424	45	4364	70	7477	95	9813
21	1527	46	4491	71	7593	96	9866
22	1631	47	4618	72	7708	97	9913
23	1737	48	4745	73	7822	98	9952
24	1845	49	4873	74	7934	99	9983
25	1955	50	5000	75	8045	100	

The

The Uſe of this Table is as follows.

1. It is Requiſite that the Bung and Head Diameters, the Cask Length, the whole Content, and the Dry and Wet Inches be all known, and then if the Queſtion be

What is wanting, or what is Remaining in the Cask?

Divide accordingly either the Dry or Wet Inches by the Bung Diameter, and the Quotient ſeek in the Table under V, or Verſed Sine; againſt it ſtands a Number, which multiplyed by the whole Content exhibits the vacuity, if your Dividend were the dry Inches; or ſhews the Remaining Liquor, if your Dividend were the wet Inches.

Suppoſe the Bung Diameter 28, the Content of the Cask 60 Gallons, and Dry Inches 7.

Divide 7 by 28, by adding two Cyphers,

7ØØ(25
288
2

ſeek this 25 in the Table, over againſt it you find 1955, which Number multiply by the whole Content 60, ſo is the Ullage or Wants, 11.7300; cutting off the laſt

4 Figures is 11 Gallons, and almoſt ¾ of a Gallon.

Now if the Queſtion be, What Quantity of Liquor is remaining in this Cask?

Divide the Wet Inches by the Bung Diameter, that is 21 the Wet Inches by 28 thus, 28 | 2100(75. Now againſt 75 in the Table you find 8045, which being multiplyed by 60 the whole Content of the Cask.

The Remaining Liquor is	48.2700
The Wants was	11.7300
The whole Contents.	60.0000

Now if after Diviſion there happen a Remainder or Fraction, and that be above Half the Diviſor, I take the next bigger Number: Or if it be leſs than half the Diviſor, I take the ſame Number which is in the Quotient.

The Deſcription and Uſe of the Gauging Rule.

This Rule is commonly 4 Foot long, and is made to double in 4 joints for convenient Portage: It hath alſo 4 ſides, on which are drawn ſeveral Lines. 1. There

1. There are two Lines called Diagonal, the one for Ale, the other for Wine Meaſure; Put the end which is cut ſlope-ways in at the Bung-hole, and let it touch the bottom of the Head; and the Number at the Bung ſhews the Ale or Wine Gallons reſpectively: This will give a very good eſtimate of the Content of all Cask in the form of a *London* Beer-Barrel, or the *French* Wine Hogſhead.

2. On another ſide there is put a Line of Inches from 1 to 48, each Inch decimally divided: And alſo upon the ſame ſide you have *Oughtred*'s Gauge Line, it being a Line of one Thirds of Area's of Circles in Wine Gallons, by which you may Gauge a Cask after this manner,

Put your Rule down at the Bung perpendicularly, obſerving what Number appears juſt even with the inſide of the Cask, ſuppoſe it 7: ſet that down twice, then take the Diameter at the Head, and let that ſhew you 6 upon the ſame Line, ſet that down to the former, add theſe three Numbers together, and multiply the Sum by the Cask length ſuppoſe 30, then cut off one place from the Product towards the Right Hand, and the Figures towards the left hand are your Number of Wine Gallons contained in that Cask.

If your Diameter fall amongſt the Diviſions between the Numbers, you muſt cut off 2 places from the Product. *Example.*

Bung Diameter	71	7	
	71	7	
Head Diameter	58	6	
	200	20	
Cask Length.	30	30	Length.
Content	60.00	60.0	Content.

3. On a third Face of this Rule is a Line of equal Parts numbred from one to 96: This Line conſidered together with that laſt before-mentioned, (*viz.* *Oughtred*'s Gauge-Line) do make a Table of Area's of Circles in Ale-Gallons; ſo that if you find your Diameter in this Line, turn up the other Face, and againſt your Diameter you ſhall have the Area of your Circle in Ale-Meaſure. As for *Example.*

The Diameter of a Circle is 19 Inches, the Area of that Circle upon the other edge in *Oughtred's* line is a little above one Gallon, ſo the Diameter 30 Inches, the Area is 2.5 Gallons, ſo the Diameter 67 inches, it holds 12.5 Gallons upon one Inch of Depth.

The uſe of theſe lines thus together is the

the ſame with that of the Table of Areas of Circle in Ale Gallons, *pag.* 124.

4. On the fourth ſide of this Rule is drawn a line of Numbers, vulgarly called *Gunters* Line, being a Line of Logarithmes, whoſe uſe I have deſcribed In Inſtrumental Arithmetick, *pag.* 37, 38, 39, 40: Only in this place take notice as to Gauging, that at 17.2 you have a ſmall Braſs Pin, whereon to ſet the foot of your Compaſſes, and is called the Gauge point for Wine Gallons marked *W. G.* And another ſmall Braſs Pin at almoſt 19 the Gauge point for Ale Gallons, marked *A. G.* by which they are eaſily known. Now to Gauge a Cask by this line, you muſt firſt find the Diameter at the Head and Bung, and alſo the Casks length by the line of Inches: Theſe being had, find your Mean Diameter, by adding double the Bung Diameter, to once the diameter at the Head, and divide their Sum by 3, the Quotient take for your Mean diameter: Then with your Compaſſes ſet one foot in the Gauge point, whether it be for Wine or Ale reſpectively, and Extend the other to the mean diameter upon your Line of Numbers, and keeping the points at that diſtance, ſet one foot at the Number, expreſſing the Casks Length, and from thence double the diſtance of the feet of Compaſſes, exhibits the Content in Ale-

or Wine Gallons respectively: As for *Example*, Bung diameter 27, Head diameter 24, Length 30 Inches, the mean diameter will be 26, which in Ale Gallons 57, in Wine Gallons 69, and having the mean diameter of a Conical Tun (28½) depth of Liquor (29) The Quantity of Ale Gallons will be found (63½) after the same manner, and is an Approximation near enough the Truth for common practice.

```
 27
 27
 24
---
 78(26
 33
```

5. There is also another Line that runs parallel with this Line of Numbers, and is called a Line of Segments, but I do not like the Hypothesis upon which it is framed; the way by the Table of Areas of Segments being more perfect; However it being upon the Rule, I shall shew you how to find the Ullage or wants in a Cask by this Line and the Line of Numbers.

Suppose the Bung diameter 24 Inches, wet 18 inches. Content 50 Ale Gallons, What is the Ullage or Wants in this Cask?

As 24 on the Line of Numbers is to Radius on the Segments, so is (6) the dry inches on the Line of Numbers to 17. 8 on the Segments. Then as Unity to 50 on the Line of Numbers, so is 17.8 on the Numbers to 8. 9 Ale Gallons, the wants required. So much for Gauging.

CHAP.

CHAP. XIV.

NOW beſides this kind of meaſuring a Triangle, by taking the whole Content both of a Superficial and Solid Triangle; There is another way of Meaſuring a Triangle, or of finding the length of ſome of its ſides, or the Quantity of ſome of its Angles by other Sides and Angles given or known.

For every Triangle conſiſting of 3 ſides and 3 Angles, any 3 of theſe 6 parts being known, the reſt may be eaſily found, either firſt Geometrically by Protraction, *pag.* 85, 86, 87: Or Secondly, Arithmetically, by the Rule of Proportion, as it is performed Logarithmetically, *pag.* 31, and Inſtrumentally, *pag.* 41. This I call properly the Reſolution of a Triangle, or Trigonometry.

Many Learned Authors have made this very particular the ſubject of whole Books, as *Petiſcius*, *Norwood*, and others; But for my preſent Purpoſe, I ſhall content my ſelf with theſe following neceſſary Rules and Directions, which well applyed, contain in ſhort the whole Art of Trigonometry.

OF TRIGONOMETRY.

SECT. 1.

First then, By *Protraction*, or by *Scale* and *Compaſs*.

PROBLEM 1.

Having any 2 Angles and a Side included or contained betwixt them, to find the other Two Sides, and the Third Angle.

AS ſuppoſe the Angles given in the following Triangle were the right Angle at the Baſe 90 degrees, and the Angle at the Hypothenuſe 30 degrees, and the Baſe included betwixt theſe two Angles 400. To find the length of the Hypothenuſe,

nuſe, and the length of the Perpendicular, as alſo the third Angle at the Perpendicular.

Firſt, From a line of equal Parts take in your Compaſſes the Length of the Baſe 400, and drawing a Line at pleaſure, prick the ſaid diſtance upon the ſaid line: And at one end of the Baſe protract an Angle of 30 degrees DE: At the other end of the ſaid Baſe protract another Angle of 90 degrees, as FG, and drawing the perpendicular infinitely, and alſo the Hypothenuſe, where theſe two interſect, or cut one another, doth confine the length of both the ſides meaſured upon the ſame Line of equal Parts with your Compaſſes; and alſo doth include the Third Angle ſought betwixt them, *viz.* 60 degrees.

PROBL. 2.

Having Two Sides, and the Angle included betwixt them, to find the other Two Angles, and the Third Side.

This you will find not to differ much from the former: Only firſt you muſt Protract the Angle, drawing the ſides infinitely, and afterwards from the Angular point prick out either way their Exact Length, and from thoſe Points draw the third ſide,

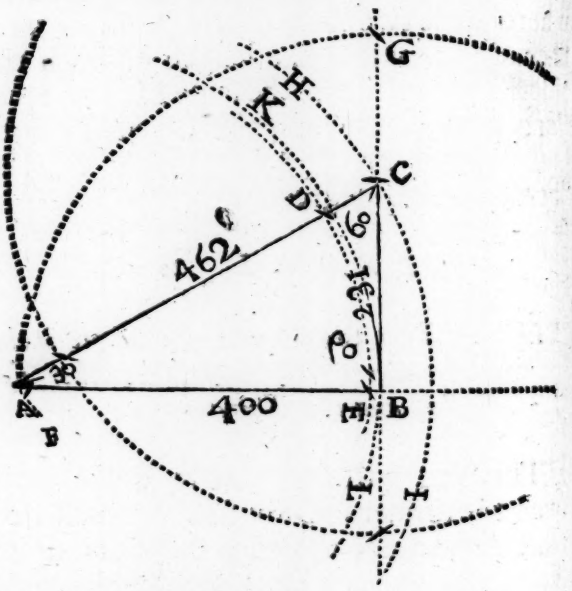
which

which will include at either end the Two Angles ſought.

PROBL. 2.

Having the Three ſides given, to find the Three Angles of any Right Lined Triangle.

Firſt, Take the diſtance of the longeſt ſide in your Compaſſes, and from any point as A, with that ſame diſtance draw an Arch of a Circle H I; then take the ſide



next the longeſt, and from the ſame point draw another Arch K L. Laſtly, from any

any point in one of these Arches, to any other point in the other Arch, prick the length of the Third side, and drawing Lines from these three points from one to another, these Lines or Sides will include the three Angles required, A. B. C. A containing 30 degrees, B 90 degrees, and C 60 degrees.

This way of finding Sides and Angles by Geometry or Protraction, though it is not so exact as by Arithmetical Operation; yet it is sufficient for ordinary Use, and of all other, both the most easie and demonstrative, and also the most pleasant and delightful.

Note, That in every right Angled Triangle, if you have one of the acute Angles given, the other is also given, because it is the complement thereof to 90 degrees as the complement of 30 is 60. Also, if in any right Lin'd Triangle whatsoever you have any two of the Angles given, you have also the third Angle given, because the three Angles of any right lin'd Triangle, is equal to a Semicircle, or 180 degrees.

SECT.

SECT. 2.

But in the Second Place, by ſome ſides or Angles of a Triangle given, to find the Reſt, (which is properly the Reſolution of a Triangle) is the moſt exactly performed by Arithmetick.

PROBL. 1.

Firſt, By Common Arithmetick, Sides may be found by theſe Rules.

The Square of the Baſe and Perpendicular is equal to the Square of the Hypothenuſe: As ſuppoſe the Baſe to be 4, the Square of 4 is 16; then ſuppoſe the perpendicular 3, the Square of 3 is 9; theſe 2 Numbers added together is 25: Now the Hypothenuſe being 5, the Square of 5 is alſo 25. Likewiſe having the Hypothenuſe, and one of the other ſides, you may find the other ſide: As ſuppoſe the Hypothenuſe 5, the Square of which is 25; as 5 multiplyed by 5 is 25: Then ſuppoſe the other ſide given be the Baſe 4, the Square of that multiplyed in it ſelf is 16: This being deducted from the Square of the Hypothenuſe 25, there Reſts 9, which is the Square of the perpendicular 3: The ſame Rule to be obſer-

obſerved if the Perpendicular had been given, and the Baſe required.

SECT. 3.

Having ſhewed you how to find Sides by *Common Arithmetick*, I ſhall proceed to ſhew you, How both Sides and Angles may be found by *Logarithmetical* and *Inſtrumental Arithmetick* by theſe following Rules. The firſt is more particular for Right Angled Triangles only, the other two more general for all plain Triangles whatſoever.

RULE 1.

1. *Firſt then, having Two Sides, and a Right Angle betwixt them given, To find either of the Two Angles, and the Third ſide. The Proportion is,*

As the greater ſide given is to the leſſer ſide, ſo is the Tangent of 45 degrees to the Tangent of the leſſer Angle, and its complement to 90 is the greater Angle: As for Example, in the former Triangle, *pag.* 134 : The Baſe being given 400, and the perpendicular 231, and the Right Angle 90 contained betwixt them : Then ſay by Logarithmes, as the Logarithme of 400 is to the Logarithme of 231, ſo is the

Tangent

Tangent of 45 to the Tangent of the leſſer Angle, (*viz.* 30 degrees) which deducted from 90, leaves 60 degrees for the other Angle: This is done by working by the *Rule of Three* in *Logarithmes*, as is taught *page* 31, but I ſhall give you another Example in this place.

If 400 give 231, what gives Tangent 45?

The Logarithme of 400 is	2.602059
The Logarithme of 231 is	2.363612
The Tangent of 45 is	10.000000
Theſe 2 added together is	12.363612
From whence deduct the firſt	2.602059
Reſts the Tangent of 30 deg.	9.761553

for the leſſer Angle: The complement whereof to 90 degrees, is 60 the greater Angle.

The ſame may be done by Inſtrumental Arithmetick, or the Lines of Numbers and Tangents by the Rule of Three upon thoſe Lines as is taught *pag.* 41; but I ſhall here add this Example more, Extend the Compaſſes from 400, to 231 in the Line of Numbers, the ſame Extent upon the Line of Tangents

Tangents will reach from 45 to 30 degrees for the leſſer Angle: the greater Angle being its complement to 90 *degrees, viz.* 60: And the two Angles found, and the two ſides given, you may eaſily find the third ſide by the Rule in *Common Arithmetick, pag.* 136, or by the third Rule following, to which I refer you.

RULE 2.

The ſecond Rule for reſolving of a Triangle is more general than the former, becauſe it reſolves not only a right angled Triangle, but alſo an Acute, and an Obtuſe Angled Triangle, that is any Right Lin'd Triangle whatſoever, and it is this.

As the Sum of two of the ſides is to the difference of the ſame ſides, ſo is the Tangent of half the Sum of the oppoſite Angles to the Tangent of half their difference.

Which I ſhall more fully explain by this Example, As ſuppoſe two ſides to be given, and any Angle betwixt them, to find either of the other Angles: Firſt, Add the ſides together, which ſuppoſe 303 and 176 makes 479: And Secondly, Subſtract the one from the other, their difference is 127. Now ſuppoſe the Angle given to be 110 *d.* 30 *m.*

Take

Take the complement thereof to 180, which is 69 *d.* 30 *m.* and this is the Sum of the 2 unknown Angles, (becauſe every right lin'd Triangle is equal to 2 right Angles, or 180 degrees.) Now the half of this 69*d.* 30 *m.* is 34*d.*45*m.* this being done, the Proportion is,

As the Logarithme of the whole Sum 479 is to the Logarithme of the difference 127, ſo is the Tangent of the half Sum of the two unknown Angles, *viz.* 34 *d.* 45 *m.* To the Tangent of half the difference between the ſaid Angles, *viz.* 10 *d.* 25 *m.*

For the ſaid 10 *d.* 25 *m.* being added to the half Sum (34 *d.* 45 *m.*) the Sum will be 45 *d.* 10 *m.* the quantity of the bigger Angle; and being ſubſtracted from the ſame, will ſhew the Quantity of the leſſer Angle, *viz.* 24 *d.* 20 *m.*

The Proportion may be wrought either by the *Rule of Three* in Logarithmetical; or by the Lines of Numbers and Tangents in *Inſtrumental Arithmetick*, *pag.* 31. and 41.

RULE 3.

The third General Rule, which is the laſt, I ſhall lay down, but not the leaſt, is the moſt general of all the reſt, and indeed may ſerve inſtead of all; and by me has been accounted even all in all; inſomuch as I once thought to have given no other, and but only for varieties ſake there is no other abſo-

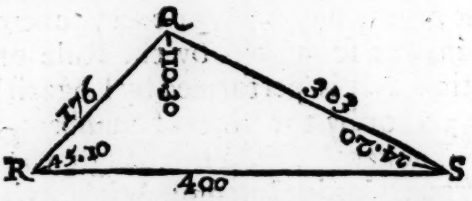
absolutely necessary, which for its excellency deserves to be writ in Letters of Gold, which together with the Golden Rule or Rule of Three, as it is wrought and performed by Logarithmes, *pag.* 31, and by the lines of Numbers, Sines and Tangents, *pag.* 41, is sufficiently able to do whatsoever is necessary in Trigonometry, which take as follows.

In all Triangles whatsoever, every side is proportional to its opposite Angle, and every Angle to its opposite side: And further, as the Angle opposite to one side is to the Angle opposite to the other side, so is the sides themselves to one another, *& è contra*, the sides are to the Angles.

As for Example in the Triangle, *pag.* 134. As the Angle A 30 *d.* is to the perpendicular 231, so is the Angle C 60 *d.* to the Base 400, and so is the Radius or the right Angle 90 *d.* to the Hypothenuse 462, *& contra*, so that there needs no more but having any Three of the six parts of a Triangle given, any one, or every one of the rest may be found out by the Rule of Proportion, as it is performed by Logarithmes, *pag.* 31. or by the Lines of Numbers, Sines, and Tangents, *pag.* 41. though the Rule of Proportion in Common Arithmetick will not do it in Common Numbers, that is, will not find Angles. How to find Sides by Common

mon Arithmetick is taught before, *pag.* 136, But by Logarithmes or the Lines you may find not only Sides, but Angles alſo: Taking this obſervation along with you, That when you meet with an Obtuſe Angle, to find its oppoſite ſide, you muſt work by its complement to 180, which being uſed inſtead of the Angle it ſelf, will effect the ſame thing in every reſpect. As to give an example in the Obtuſe Angled Triangle QRS, whoſe Angle at Q is obtuſe, being 110 *d.* 30 *m.* the Angle *R* 45 *d.* 10 *m.* the Angle *S* 24 *d.* 20 *m.* the ſide RS oppoſite to the obtuſe Angle is 400, the ſide QS 303 oppoſite to the Angle R; The ſide QR 176 oppoſite to the Angle S, ſo the proportions ſtand thus,

As 24 *d.* 20. *m.* the Angle at S is to 176, ſo is 69 *d.* 30 *m.* (the complement of 110 *d.* 30 *m.* to 180) to 400: And ſo is 45 *d.* 10 *m.* to 303, *& è contra*: Therefore ſay by the Rule of proportion, either by Logarithmes, or by Lines:



If 24 *d.* 20 *m.* give 176, what gives 69 *d.* 30 *m.* or 45 *d.* 10 *m.* to find the ſides. Or,

If

If 176 give 24 *d.* 20 *m.* What gives 400, or 303 to find the Angles: ſee *pag.* 31 and 41, where you have the way of working theſe, or any ſuch like proportions by the *Rule of Three*, either in Logarithmes, or by the Lines.

CHAP. XV.

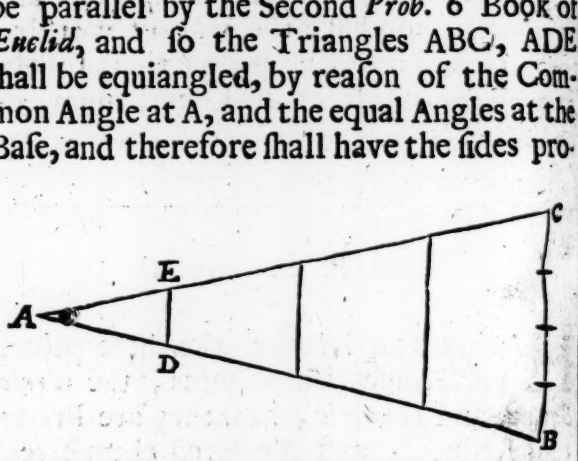
HAving now handled the three principal Geometrical Figures, the *Circle*, *Square*, and *Triangle*; how they are Produced, Reduced, and Meaſured themſelves; and conſequently all other Figures or Bodies by their Rules; To conclude, What I have to ſay in reference to Geometry in general, ſhall be to ſhew you the ſeveral excellent uſes of Mr. *Gunters* Sector, which being properly Geometrical, I reſerved for this place, to ſhut up general Geometry.

SECT. 1.

Firſt then, To ſhew the Ground of the Sector.

Let AB, AC repreſent the Legs of the Sector: Then ſeeing theſe two, AB, AC are equal, and their Sections AD, AE alſo equal, they ſhall be cut proportionably, and

if we draw the Lines BC, DE, they will be parallel by the Second *Prob.* 6 Book of *Euclid*, and ſo the Triangles ABC, ADE ſhall be equiangled, by reaſon of the Common Angle at A, and the equal Angles at the Baſe, and therefore ſhall have the ſides pro-



portional about thoſe equal angles, by the fourth *Prob lib.* 6. *Euclid.* Again the ſide AD ſhall be to the ſide AB as the Baſe DE unto the parallel Baſe BC, and by Converſion AB ſhall be unto AD, as BC unto DE. And by permutation AD ſhall be unto DE, as AB to BC, *&c.* ſo that if AD be the fourth part of the ſide, AB, then DE ſhall be the fourth part of his parallel Baſis BC, the like reaſon holdeth in all other Sections. And by the way, to underſtand the general aim and uſe of the *Sector*, it is to perform the ſame by Lines, which Arithmetick does by Numbers, as I ſhall ſhew you particularly: Now theſe lines which are found out by the Sector, are of two ſorts: Either firſt Lateral,

Lateral (or ſuch as are found upon the ſides of the Sector) as are the Lines, AB, AC: Or Secondly, Parallel ſuch as are the lines CB, DE from one ſide of the Sector to the other in its parallel points.

Having premiſed thus much in general, I ſhall ſhew you in particular the ſeveral uſes of the *Sector*.

Now you muſt underſtand that (beſides the Lines of Numbers, Sines, and Tangents upon the outward edge of the Sector, and a foot divided into 12 inches; as alſo another line of equal parts, or a foot divided into 100, or to be ſuppoſed 1000 parts upon the Inner edge of the Sector.) There are theſe ſeveral Lines upon the Sector it ſelf.

Theſe four, as alſo the 2 laſt are the principal Lines.
1. A Line of Sines.
2. A Line of Tangents.
3. A Line of Superficies.
4. A Line of Solids.

Theſe 5 Lines are leſs principal, and for particular Uſes.
5. The Line of Quadrature.
6. The Line of Segments.
7. The Line of inſcribed Bodies.
8. The Line of Equated Bodies.
9. The Line of mettals.

The other two Principal Lines.
10. The Meridian Line.
Laſtly, The Tangent Line.

And of theſe in the ſame order I here Rank or Reckon them, and firſt of the firſt (to wit) The Line of Sines.

SECT. 2.

The Line of Sines.

1. To Protract Angles by a Line of Right Sines is taught *pag.* 87, 88. And to avoid Tautology, I ſhall not here repeat it, but refer the Reader thereunto, being one excellent uſe of this Line.

2. If the Radius of a Circle be the ſame with the Lateral Radius or Semiradius of the Sector, the reſpective ſeveral Sines are the ſame as in this Line of Sines: But if it be either greater or leſſer, ſet over the given Radius in the Line of Sines betwixt 90 and 90, and the Parallel Sines all along are the right Sines to the ſame Radius; and hereby you may divide any Line given, as a Line of Sines; And by the ſame Rule any ſine being given, you may find the Radius thereof, by making it a Parallel Sine in its reſpective Number, the Radius will be betwixt 90 and 90.

3. To find the Chord of any Ark, obſerve that the Right Sine of the ſame Ark is half the Chord required; as the Sine of 10 is the Chord of 20, the Sine of 20 is the Chord of 40, *&c.* all along. The Radius or 60 degrees of a Line of Chords being ſet over as a Parallel betwixt 30 and 30

of

of the Line of Sines upon the Sector, mark this well.

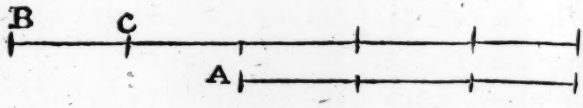
4. To find the verſed Sine of any Ark. If you reckon from 90 at the end of the Sector to 80 towards the Center, that diſtance is the verſed Sine 10; from 90 of to 70 the verſed ſine of 20, and ſo of the reſt. if the verſed Sine required exceed a Quadrant, as ſuppoſe 130 degrees, Firſt, Take the whole *Radius*, and as many degrees as it is above 90, (as in this Example it is 40) add thereunto from the end of the Sector towards the Center.

SECT. 3.

Of the Line of Lines.

This ſerves for a Line of equal parts upon any occaſion, as they are numbred on the ſides of the *Sector*, if the Radius be the ſame with the Lateral Radius. But if you would divide any other Line given into equal parts, ſet it over as a Parallel Radius betwixt 100 and 100, and the Parallel diſtances ſhall divide the ſame given Line into the ſame equal parts, according to the number which the points of your Compaſſes do ſtay upon: As if you would take or mark out 40 of thoſe parts, the parallel diſtance betwixt 40 and 40 markt out upon the Line given, is 40 of the ſaid parts, and ſo of the reſt.

Or, Suppoſe the Line B were to be divided into 5 parts, Firſt, I take the Line B betwixt the Compaſſes, and to it open the



Sector in the point 5 and 5, ſo the parallel between the point 1 and 1 doth give me the Line BC, which doth divide the ſame into the parts required.

PROBL. 1.

To increaſe or Diminiſh a Line in a given Proportion.

As let A be a line given to be increaſed in the proportion, as 3 to 5. Firſt, I take the line A in the Compaſſes, and open the Sector till the points of the Compaſſes do reach juſt between 3 and 3 on the ſides of the Sector; ſo the Parallel between the points 5 and 5 doth give me the Line B, which was required. In like manner, if B were to be diminiſhed as 5 to 3, Firſt, ſet over B betwixt 5 and 5, and its parallel betwixt 3 and 3 gives you A diminiſhed as 5 to 3.

PROBL. 2.

To find a proportion betwixt two or more right Lines given.

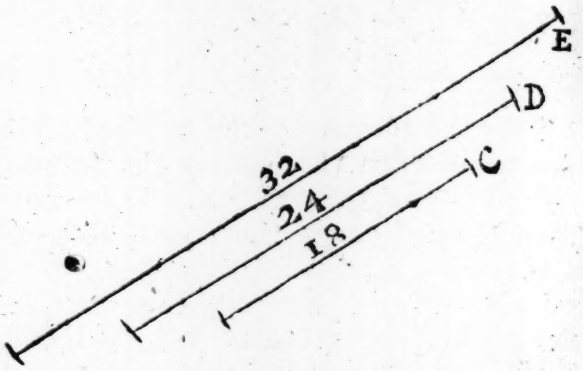
Take the greater Line given, and according to it open the *Sector* in the Points of 100 and 100; then take the leſſer Lines ſeverally, and carry them parallel to the greater till they ſtay in like points; ſo the number of Points wherein they ſtay ſhall ſhew their Proportion to 100, ſo the line B is in Proportion to A, as 100 to 60, and A to B, as 60 to 100.

PROBL. 3.

Two Lines given, to find a Third in continual Proportion.

Take the two Lines or Numbers given, and ſet them upon both ſides of the Line of Lines (or equal Parts) from the Center, and Mark the Terms to which either of them extendeth; as ſuppoſe the Line C 18, and D 24 were the Lines or Numbers given. Meaſure with your Compaſſes the ſecond Line D 24, and open the *Sector* at that diſtance in the Terms of the firſt Line or Number C, and ſo keep the *Sector* at this

Angle. The Parallel diſtance between the Terms of the ſecond Lateral Number or



Line (24) being meaſured in the ſame Scale from whence his Parallel was taken, will give the third number proportional, *viz.* the Line E, containing 32 of the ſame equal parts.

PROBL. 4.

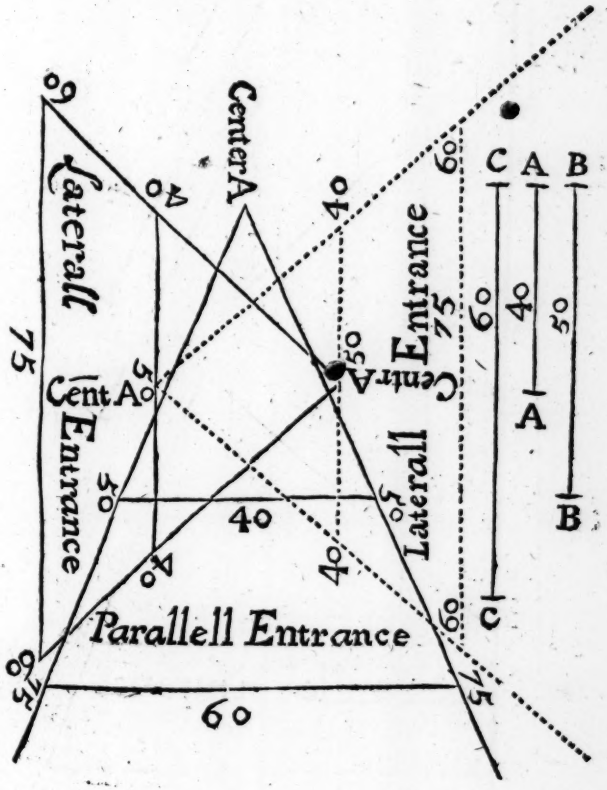
Three Lines or Numbers being given, to find a Fourth.

Let the Lines given be A 40, B 50, C 60, and the Queſtion be this, If 40 Moneths give 50 *l.* What ſhall 60 Moneths give?

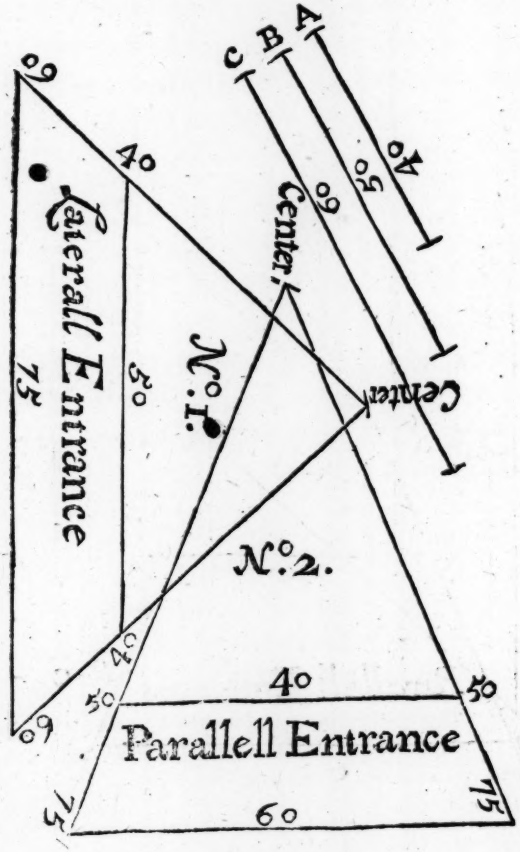
This you ſee is perfectly the Rule of 3, in

in the ſolution whereof a principal uſe of the *Sector* doth conſiſt.

The firſt and third Lines or Numbers, (*viz.* A. 40. C 60.) muſt be placed on both ſides the *Sector* from the *Center*, and with the ſecond, *viz.* B 50. open the *Sector* in the



Terms of the first, *viz.* A 40. And the Parallel distance between the terms of the third, *viz.* C. 60, shall be the fourth proportional, to wit 75, as in the Example; and this is called Lateral entrance.



But

But if you would enter with a Parallel, do thus. First, with the first Line or Number A 40 in your Compasses open the *Sector* to that distance in the terms of the Second, *viz.* B. 50 (or make it the distance between 50 and 50 on the sides of the Sector.) Then take with your Compasses the third Line or Number C 60, and with that distance observe where the Points of the Compasses do stay in the Parallels, and that is the fourth proportional, which you will find 75, as before.

SECT. 4.

The Use of the Line of Lines, together with the Line of Sines, may be these.

PROB. 1.

In TRIGONOMETRY

Any Two Sides and an Angle included given, to find the Third Side.

Let the *Sector* be opened to the Angle given (30 *d.*) and observing the length of either side upon the sides of the Sector in the Line of Lines, the parallel distance from one point to the other, where the sides of the Triangle terminate, doth give the side

required. As ſuppoſe the Baſe of the Triangle *pag.* 134, be one ſide given, *viz.* 400, the Hypothenuſe the other ſide 462, the Angle included 30 at A, you will find the diſtance with the Compaſſes from 400, to 462 to be 231 the ſide required.

PROBL. 2.

Three ſides being given, to find an Angle.

Let the two containing ſides be laid on the Lines of the Sector from the Center one on one line, and the other on the other; and let the third ſide which is oppoſite to the Angle required be fitted over in the termes of the other two; ſo ſhall the *Sector* be opened in thoſe Lines to the Quantity of the Angle required. As for *Example* in the before-mentioned *Triangle, pag.* 134.

Having

Having one of the containing ſides 400 upon one ſide of the Sector, and 462 on the other, and ſetting theſe at the diſtance 231, the Angle at the Center will be found 30 *d.* Or ſuppoſe one ſide 72 in the Line of Lines, the other 54, and fit the oppoſite ſide (to the Angle required) over between 72 and 54; then with your Compaſſes meaſure the diſtance betwixt 50 and 50 in the ſame Line of Lines, and the ſame diſtance meaſured in the Line of Sines ſhall ſhew half the quantity of the Angle required.

As ſuppoſe the three ſides given be AD 100, AC 75, AB 35. I might take 35 for the ſide CD out of the Line of Lines, and ſet it over from 100 to 75. This done, I take the diſtance between 50 and 50, and meaſuring it in the Line of Sines, I find it to be 8 *d.* 8 *m.* the double whereof is 16 *d.* 16 *m.* the Angle required.

SECT. 2.

The Uſe of the Line of Lines, together with the Line of Sines in Navigation, *ſee in its proper place.* That is,

1. To know how many Leagues do anſwer

ſwer to a Degree of Longitude in every ſeveral Latitude.

2. To know how many Leagues do anſwer to a Degree of Latitude in every ſeveral Rhumb. *Vide ut ſupra.*

SECT. 5.

The Uſe of the Line of Lines, together with the Line of Superficies in Extracting the Square Root.

1. The Line of Superficies is ſaid to be divided into 100 parts; therefore hath upon it twice the Figure 1, the former ſignifying Units, the latter Tens in that reſpect of the Line being divided into 100 parts.

2. Obſerve, If the Number, whoſe Square Root you deſire, conſiſt of even Figures, as 2, 4, 6, 8 Figures, or the like, the Number given ſhall fall out between the tenth diviſion and the end of the Sector, (or which is all one) between the latter Figure of 1, and the end of the Sector: But if the Number) whoſe Square Root you deſire) be odd, that is 1, 3, 5, 7 Figures, or the like, the Number given ſhall fall (or muſt be meaſured (between the Center of the *Sector*, and the *Tenth Diviſion.*

3. Theſe

3. Theſe being obſerved, ſet one foot of your Compaſſes in the Center, extend the other to the Term of the Number given in one of the Lines of Superficies, (which ſuppoſe 36.) The ſame diſtance applyed to one of the Line of Lines, ſhall find 6 for the Square Root.

Likewiſe to Square a Number given, firſt Extend the Compaſſes upon the Line of Lines to the Number you would Square (as ſuppoſe 6,) and apply the ſame diſtance to the Line of Superficies, will give (36) as before.

SECT. 6.

By the Line of Lines, and Solids, to find the Cube Root.

In finding a Cube Root of a Number given, prick the firſt Figure, and after that every third Figure from the right hand to the left; and if the laſt prick to the left hand fall under the laſt Figure, the Number given ſhall be reckoned at the beginning of the Line of Solids, from the Center to the firſt 1, and the firſt Figure of the Root ſhall be alwaies either 1 or 2.

If the laſt prick fall under the laſt Figure but One, Then the Number given ſhall be reckoned from the Center to the

ſecond 1, and the firſt Figure of the Root ſhall be alwaies either 2, 3, or 4.

But if the laſt prick fall under the laſt Figure but two, then the Number given ſhall be reckoned from the Center between the laſt Figure of one, and the end of the *Sector*: For you muſt know, as the Line of Superficies is ſuppoſed to be divided into 100 parts, and ſo having 2 Figures of 1 upon it, the firſt ſignifying Units, the ſecond Tens; ſo the Line of Solids is ſuppoſed to be divided into 1000 parts, and hath three Figures of one upon it, the firſt ſignifying Units, the ſecond Tens, and the Third Hundreds; ſo that according to this Diviſion the whole Line is 1000 as before. Theſe things being obſerved, the Rule for finding the Cube Root is this.

Set one Foot of your Compaſſes in the Center of the *Sector*; Extend the other in the Line of Solids to the point of the Number given according to the Rules before given: This diſtance applyed to the Line of Lines, ſhall ſhew the Cube Root of that Number.

Or to Cube a Number, Firſt, Extend the Compaſſes upon the Line of Lines; and then meaſure the ſame diſtance upon the Line of Solids, it will give you the Cube Number of a Cube Root given.

If the Reader desire further concerning these two last propositions of finding the Square and Cube Root by the Sector, I refer him to *Gunter* himself upon this Subject.

SECT. 7.

Of the Line of Superficies alone or single: As also the Line of Solids alone or single.

Observe first, That the Line of Superficies and Solids do serve to find Proportions betwixt Square and Cubique Bodies: As the Line of equal parts do betwixt lines (from thence called properly the Line of Lines) therefore the same Rules may in a manner serve for these which are given for the other right applyed, *mutatis mutandis.*

Secondly observe, That whatsoever is said concerning Superficies, the very same Rules serve for Solids, only that they are wrought upon different Lines. Therefore to avoid Repetition, observe to apply these Rules given for Superficies to Solids also, *mutatis mutandis* as I said before.

PROBL.

PROBL. 1.

Therefore to proceed to find a Proportion between two or more like Superficies (as is before done in Lines,) do thus.

Take one of the ſides of the greater Superficies, and according to the ſaid ſide given open the *Sector* in the points of 100 and 100, then take the like ſide of the leſſer Superficies, and carry it parallel to the former, till they ſtay in like Points, ſo the Number of Points wherein they ſtay ſhall ſhew the proportion to 100 in the Lines of Superficies: As for Example, Let A and B be the ſides of like Superficies. Firſt, I take the ſide A in the Compaſſes, and to that diſtance open the Sector in the points of 100 and 100: Then keeping the Sector to that Angle, I enter the leſſer ſide B parallel to the former, and find it to croſs the Lines of Superficies in the point 40 and 40, therefore the Proportion of the Superficies, whoſe ſide is A, to that whoſe ſide is B, is as 100 to 40, or in leſſer Numbers, as 5 to 2. This Propoſition may alſo be wrought by 60, or any other Number that admits ſeveral Diviſions.

It may alſo be wrought without opening the Sector, for if the ſides of the Superfi-

cies given be applyed to the Lines of Superficies, beginning alwaies at the Center of the Sector, There will be ſuch proportion found between them, as between the Number of parts whereon they fall.

What is ſaid of Superficies, the like is to be underſtood of Solids wrought upon their own Lines.

PROBL. 2.

To Augment or Diminiſh a Superficies in a given proportion; or (which is the ſame) To add one Superficies to another, or ſubſtract one from another, will be all underſtood by this Example following.

Suppoſe A and B were the ſides of two Squares, or the Diameters of 2 Circles, and it were required to make a third Square or Circle equal to them both.

Firſt, The proportion between A and B will be found to be as 100 to 40, or in leſſer numbers, as 5 to 2; then becauſe 5 and 2 added together make 7, I augment the ſide A in proportion of 5 to 7, and produce the ſide C, on which if I make a

Square, or make it the Diameter of a Circle, the ſaid Square or Circle ſhall be equal to both the Squares of A and B, or to two

Circles

Circles made upon their Diameters, which was the thing required. To ſubſtract one
A from another is after the ſame manner, as from 5 take 2, the Remainder is 3, ſo it muſt be diminiſhed in proportion of 5 to 3, which is D.

PROBL. 3.

To find a Mean proportional between 2 Lines, which may be ſuppoſed the ſides of a Square, or Diameters of a Circle, and thereby to find ſeveral Corollaries.

Suppoſe the Lines given be A A and
A AC. Firſt find the proportion betwixt them as they are Lines as is taught *pag.* 60. which will be found to be as 4 to 9, wherefore I take the Line A C, and put it over to the Lines of Superficies between 9 and 9, and keep-
B ing the Sector at this Angle, his Parallel between 4 and 4, gives me A B for the mean Proportional: Or take the Line AA, and put it over in the Lines of Superficies between 4 and 4, and its Parallel between 9 and 9, will give AB for a Mean Proportional, which upon the ſame Scale of equal Parts will be found 6; for as 4 is to 6,
C ſo is 6 to 9.

Upon

Upon finding out this Mean Proportional depend many Corollaries, especially these following.

First to make a Square equal to any Superficies given.

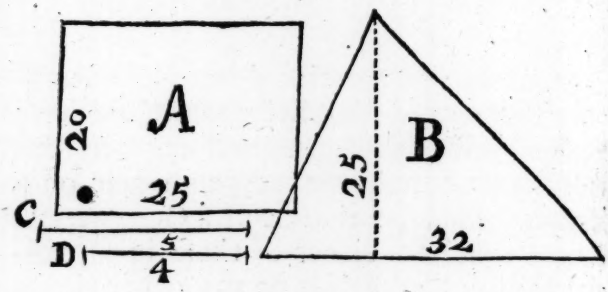
As a mean proportional between the unequal sides of a Parallelogram (or long Square:) or a mean proportional between half the Base, and the perpendicular of a Triangle shall be the side of a Geometrical Square equal to either of these in superficial Content. And if it be any other right-Lin'd Figure, it may be resolved into Triangles, and so a side of a Square found equal to every Triangle, and these reduced to one equal Square, it shall be equal to the whole Right-Lin'd Figure given.

Secondly, To find a Proportion between several Superficies though unlike to one another.

As for Example, The Oblong A, and the Triangle B are Superficies unlike to one another, therefore to find a Proportion between these.

First, Between the sides of A, *viz.* 20, and 25, I find a mean proportional, as C. This is the side of a Square equal to A, then between the perpendicular of the Triangle B, and half his Base, I find a mean proportional,

portional, *viz.* D. This is the ſide of a Square equal to B.



But the proportion between the Squares of C and D will by the Rule for it *pag.* 160 be found to be as 5 is to 4, *vide* the Rule it ſelf.

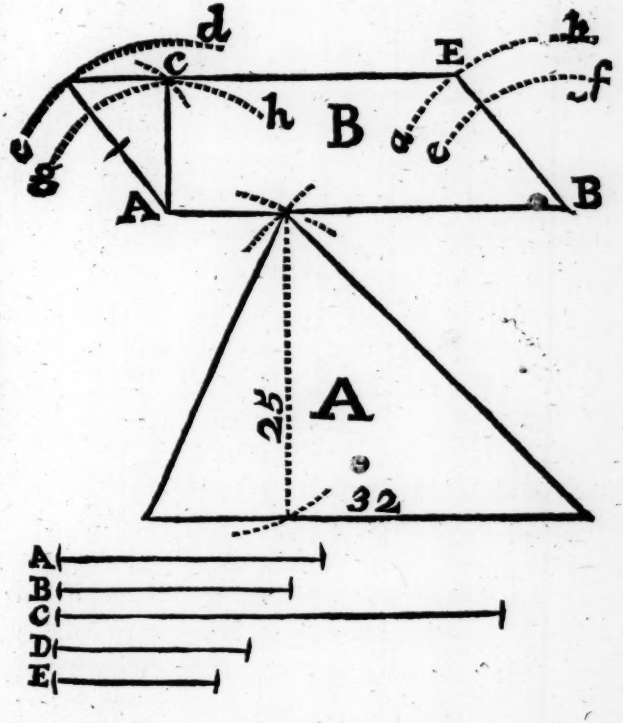
Thirdly to make a Superficies like to one Superficies, and equal to another.

Let one be the Triangle A, the other the Rhomboides B, Firſt, between the Perpendicular and the Baſe of B I find a mean proportional; and note it in B, as the ſide of a Square equal to B the Rhomboides. Then between the perpendicular and half the Baſe of the Triangle A find a mean proportional, and note it in A as the ſide of his equal Square.

Wherefore as the ſquared ſide of B is to the ſquared ſide of A, ſo ſhall the longer

ſide

ſide of the Rhomboides given AB be to the longer ſide of the Rhomboides required, *viz.* C; and ſo ſhall the ſhorter ſide of the



Rhomboides given AD be to the ſhorter ſide of the Rhomboides required, *viz.* D. And ſo ſhall the perpendicular of the Rhomboides given AC be to the perpendicular of the Rhomboides required, *viz.* E. and having

ving the ſides and perpendicular, you may frame the Rhomboides up, and it will be equal to the Triangle A, which was the thing required.

And becauſe here is mention made of protracting or framing up a Rhomboides, having the longer ſide, ſhorter ſide, and perpendicular given, I ſhall here ſhew you how to do it.

Firſt then, Draw the length of the longer ſide, or the Line AB in the former Example. And then taking the Length of the ſhorter ſide in your Compaſſes, ſetting one foot in B, draw an Arch of a Circle *a* 6, as alſo ſetting one foot of your Compaſſes in A, draw the Arch *c d*. Then take the length of the perpendicular in your Compaſſes, and ſetting one foot in B, again draw the Arch *e f*. and removing to *A*, draw the other Arch *g h*. Theſe 4 Arches being drawn, lay your Ruler ſo as juſt to touch the Arches *e f* and *gh*, and draw a parallel Line from the point D, where the Ruler croſſes the Arch *c d*. to the point E, where the Ruler will croſs the Arch *a b*; and laſtly from the point D draw the Line D A, and from the point E draw the Line E B, and let fall the perpendicular AC from the point A, ſo will you have a Rhomboides as was required.

SECT.

SECT. 8.

Of the Lines of Quadrature.

The Lines of Quadrature may be known upon the Sector by the Letter Q, and by their place between the Lines of Sines.

Q ſignifies the ſide of a Square, 5 the ſide of a Pentagon, with 5 equal ſides, 6 of an Hexagon, that is, a Figure with 6 equal ſides, and ſo 7 for the ſide of an Heptagon, 8 of an Octagon, 9 of a Nonagon, *&c.* S ſtands for the Semidiameter of a Circle, and 90 for a line equal to 90 degrees in the Circumference of the ſame Circle.

PROBL. 1.

The uſe of theſe Lines may be to make a Square equal to a Circle given, or to make a Circle equal to a Square given.

[If the Circle be given, take his Semidiameter, and to it open the Sector in the points at S; ſo the parallel taken from between the points at Q, ſhall be the ſide of the Square required: Or if the Square be given, Take his ſide, and to it open the Sector in the points at Q; ſo the Parallel taken from between the points at S, ſhall be the Semidiameter of the Circle required.

PROBL.

PROBL. 2.

To Reduce a Circle given, or a Square into an equal Pentagon, or other like Sided, and like Angled Figures.

Take the ſide of the Figure given, and fit it over in his due points, ſo the parallel taken from between the points of the other Figures, ſhall be the ſides of thoſe Figures, which being made up with equal Angles, ſhall be all equal one to the other.

Other Superficial Figures not here ſet down may firſt be reduced into a Square, and then into a Circle, or other of theſe equal Figures as before.

PROBL. 3.

In the third place, To find a Right Line equal to the circumference of a Circle, or other part thereof.

Take the Semidiameter of the Circle given, and to it (or to the ſame diſtance) open the Sector in the Points at S, ſo the parallel taken from between the points at 90, in this Line, ſhall be the fourth part of the circumference, *&c.*

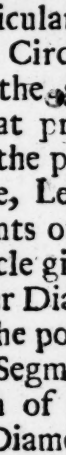
SECT.

SECT. 9.

Of the Lines of Segments.

The Lines of Segments are here placed between the Lines of Sines and Superficies, and are numbred by 5, 6, 7, 8, 9, 10.

The uſe of them is to divide a Circle given into 2 Segments, according to a proportion given, or to find a proportion between a Circle and his Segments given: For the Numbers 5, 6, 7, *&c.* do repreſent the Diameter of a Circle ſo divided into 100 parts, as that a right line drawn through theſe parts perpendicular to the Diameter, ſhall cut the Circle into two Segments, of which the greater Segment ſhall have that proportion to the whole Circle, as the parts cut have to 100: For Example, Let the Sector be opened in the points of 100 to the Diameter of the Circle given, as ſuppoſe BL, or any other Diameter; ſo a Parallel taken from the points proportional to the greater Segment required, ſhall give the depth of that greater ſegment: As if the Diameter of a Circle were BL, carry the greater ſegment LO when the Sector is opened in 100 and it will ſtay in 75, which ſhews the pro-



portion is as 75 to 100, or that the segment to the Circle is 3 parts of 4. Hereby you may find the side of a Square equal to any known segment of a Circle: For as the proportion is of the Segment to a Circle, so is the part of a Square to the Square equal to the whole Circle, which is taught before.

SECT. 10.

Of the Lines of Inscribed Bodies.

These are placed between the Lines of Lines, and may be known by the Letters D,S,I,C,O,T, of which D signifies the side of a Dodecahedron, I of an Icosahedron, C of a Cube, O of an Octahedron, and T of a Tetrahedron, all inscribed into the same Sphere, whose semidiameter is here signified by the Letter S.

The Uses are

PROBL. 1.

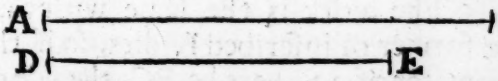
The Semidiameter of a Sphere being given, to find the sides of the 5 *Platonick or Regular Bodies, which may be inscribed in the same Sphere.*

PROBL.

PROBL. 2.

The ſide of any of the five Regular Bodies being given, To find the Semidiameter of a Sphere that will circumſcribe the ſaid Bodies.

If the Sphere be firſt given, take his Semidiameter, and to it open the Sector in the points at *S*, and the Parallel diſtance betwixt any other of the Bodies will give you the ſide of the ſame Body, which may be



inſcribed in the Sphere: As if the Semidiameter be AC, the ſide of a Dodecahedron to be therein inſcribed will be DE.

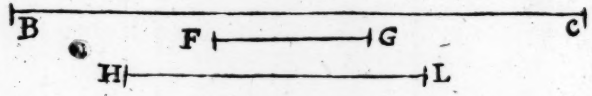
If the ſide of any other Body be firſt given, fit it over in his due points, and his parallel diſtance between the points *S* will give the ſemidiameter of a Sphere which ſhall circumſcribe the ſame Body.

SECT. 11.
Of the Lines of Equated Bodies.

Theſe are placed between the Lines of Lines and Lines of Solids, noted as the former with theſe Letters D.I.C.S.O.T, ſignifying the ſame things.

The Uſe is this,

The ſemidiameter of a Sphere given, to find the ſides of the five Regular Platonick Bodies equal to that Sphere: Or the ſides of any of the five Bodies given, to find the ſemidiameter of a Sphere, and the ſides of the other Bodies equal to the firſt Body



given. The work is the ſame with that of the former of inſcribed Bodies, ſo if the ſemidiameter of a Sphere be BC, the ſide of a Dodecahedron equal to this Sphere will be found FG, and of an Icoſaedron H L. and ſo of all the Reſt.

SECT. 12.

Of the Lines of Mettals.

The Lines of Mettals are here joyned with thoſe of Æquated Bodies, and are noted with theſe Characters, ☉ ☿ ♄ ☽ ♀ ♂ ♃, of which ☉ *Sol* ſtands for God, ☿ *Mercury* for Quickſilver, ♄ *Saturn* for Lead, ☽ *Luna* for Silver, ♀ *Venus* for Copper, ♂ *Mars* for Iron, and ♃ *Jupiter* Tinn.

The

The uſe of them is to give a Proportion between theſe ſeveral Mettals in their magnitudes and weight, according to the Experiments of *Marinus Ghetaldus* in his Book, called *Promotus Archimedes.*

PROBL. 1.

In like Bodies of ſeveral Mettals, and equal weight; having the Magnitude of the one, To find the Magnitude of the reſt.

Take the Magnitudes given out of the Lines of Solids, and to it open the *Sector* in the points belonging to the Metal given; ſo the Parallel taken from between the points of the other Metals, and meaſur'd in the Lines of Solids, ſhall give the Magnitudes of their Bodies.

PROBL. 2.

In like Bodies of ſeveral Metals and equal Magnitude, having the weight of the one, To find the weight of the reſt.

As if a Cube of Gold weighed 38 pounds, and it were required to know the weight of a Cube of Lead, having equal Magnitude: Firſt, I take 38 for the weight of the Golden Cube out of the Lines of Solids,

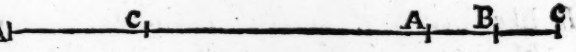
and put it over in the points of ♄ *Saturn* belonging to Lead; ſo the Parallel diſtance taken from between the points of ☉ *Sol* ſtanding for Gold, and meaſured in the Line of Solids doth give the weight of the Leaden Cube required, to be $13\frac{1}{3}$.

PROBL. 3.

A Body being given of one Metal, To make another like unto it of another Metal, and equal weight.

Let the Body given be a Sphere of Lead, containing in Magnitude 16 *d*, whoſe diameter is A A, to which I am to make a ſphere of Iron of equal weight.

If I take out the Diameter AA, and put it over in the points of ♄ *Saturn* belonging to Lead, The Parallels taken from between the joints of ♂ *Mars* ſtanding for



Iron ſhall be AB the Diameter of the Iron ſphere required, and this compared with the other Diameter AA in the line of Solids, will be found to be 23 *d*. in Magnitude; C is the length of the whole line, and from A to C again.

PROBL.

PROBL. 4.

A Body being given of one Metal, to make another like unto it of another Metal, according to a weight given.

As if the Body given were a ſphere of Lead, (or a Round Lead Bullet) whoſe Diameter is AA, and it were required to find the Diameter (of a ſphere of Iron) or of an Iron Bullet, which Iron ſphere ſhall weigh three times as much as the leaden ſphere or Bullet. I take AA and put it over in the points of ♄ which ſtands for Lead: His Parallel taken from between the points of ♂ ſignifying Iron, ſhall give me AB for the Diameter of an equal ſphere of Iron: If this be augmented in ſuch proportion as 1 to 3, it gives C for the Diameter required of a ſphere three times the weight of the former.

Having largely diſcourſed of *Geometry* in General, and of the three Famous Geometrical Figures, the *Circle*, *Square*, and *Triangle*, and having ſhewed you the Making and Meaſuring of theſe, and the Reducing of all other Figures, whether Regular, or Irregular, to ſome of theſe, if not to the two former, yet at leaſt to the laſt of theſe,

viz. a *Triangle*, and by consequence to Measure the same, by the same Rules that Measure a *Triangle* (or *Trigonometry*); Adding in conclusion the use of the *Sector*: I come now to Treat of *Geometry* according to the Proper and Genuine Etymology of the Word, which is derived from γῆ *Terra* & μέτρον *mensura*, that is, *The Measuring of the Earth*, and that (as elsewhere explained) to be understood the whole Globe of the Earth, containing *Sea* and *Land*: That which concerns the *Land* is called *Surveying*, and that which concerns the *Sea*, *Navigation*: And of these in particular. But first of Measuring this whole Globe of *Sea* and *Land* together in the General.

OF

OF THE GLOBE OF THE EARTH.

CHAP. I.

To Meaſure the whole Globe of the Earth comes under the Rules of the firſt Geometrical Figure, to wit, the Circle, and that a Solid Circle, (taking it for granted, That the Earth and Sea together make one Globular or Spherical Body.)

NOW we muſt find the ſolid Content of a Globe by the ſuperficial Content of

a Globe, and the Superficial Content of a Globe by the superficial Content of a Circle, having the like Diameter. This Diameter, we must find by the circumference of the same Circle: And lastly, This Circumference (as all other Circumferences whatsoever) must contain 360 degrees. Now if we would take the circumferences of the Earth both Sea and Land in English Miles, we must know how many of these Miles upon the Earth are answerable to a Degree in the Heavens.

In the just Quantity and Measure of a Degree, the Common Rule is much out, which accounts 5 Foot to a Pace, and 1000 paces, *i. e.* 5000 feet to a Mile, and 60 such Miles, or 300000 Feet to a Degree.

The *English* Mile, which by the Statute is somewhat more, then this will not serve the turn, which is thus, 16½ Feet make a Pole, 40 Poles a Furlong, 8 Furlongs a Mile, 60 such Miles contain 316800 Feet: By this account 1760 Yards is a Mile.

But Mr. *Norwood*'s Experiment by knowing the difference of Elevation of the Pole betwixt *York* and *London*, and measuring exactly the way betwixt them, passes for currant with all the Mathematicians I meet with of late, which was made on this manner. *June* 11. 1635. At *York*, by an Arch of a Sextan of more than 5 Foot semidiameter,

meter He took the Meridian Altitude of the Sun, which was 59 Degrees, 33 Minutes, Latitude 53 *d.* 58 *m.* *June* 11. 1633; in *London* he did the ſame, and found the Meridian Altitude of the *Sun* to be 62 Degrees, 1 Minute; in Latitude 51. 32: Now deducting the leſs latitude from the greater, there remains 2 Degrees, 28 Minutes, which is the difference of Latitude, (*York* being in 53 degrees 58 Minutes, and *London* in 51 Degrees 32 Minutes) as above noted: So by this was found 69 Engliſh or ſtatute Miles; 4 Furlongs, 14 Poles, 9 Feet in a degree; or 367200 Feet. But to make 60 *Engliſh* Miles in a Degree, each Mile ought to contain 6120 Feet: Theſe multiplyed by 60, make 367200 as above, and thoſe again by 360 is 132192000 Feet for the circumference of the whole Earth.

Now according to this obſervation of Mr. *Norwoods*, there will be according to ſtatute Meaſure 69 Miles, 4 Furlongs, 14 Poles and 9 Feet *Engliſh* in every Degree. Yet that we may not ſeem to differ ſo much from former Writers, and the Common Account we ſhall ſtill reckon, only 60 *Engliſh* Miles to a Degree, and allow ſo many more feet to a Mile, *viz.* 6120 Feet, and 2040 yards; and if we cannot allow them the name of *Engliſh* Miles, we may call them Mathematical Miles, (though no

queſtion

queſtion but our *Yorkſhire* Miles may allow of the ſame Length.) Therefore,

Firſt, 60 Miles allowed for a Degree, that multiplyed by 360, makes 21600 Miles for the whole circumference of the Earth.

Secondly, And by this Circumference to find the Diameter, Multiply 21600 by 7, is 151200, and that divided by 22, makes $6872\frac{16}{22}$ for the whole Diameter (according to the Rule, *pag.* 78.) the ſemidiameter (rejecting the Fraction) is 3436 Miles to the Center of the Earth.

Thirdly, Now having the Diameter, to find the Area or ſuperficial Content of a Circle having the like Diameter: (by the third Rule, *pag.* 78.) Multiply the Diameter by it ſelf, that is, 6872 (omitting the Fraction) by 6872, it produceth 47194384 Miles; this Multiplyed again by 11, makes 519138224, and divided by 14, the Area of the Circle is 3708130 Miles.

Fourthly, This ſuperficial Content of the Circle multiplyed by 4, (by the Rule, *pag.* 80.) produceth for the ſuperficial Content of the Globe 148325204 Miles. And,

Fifthly, For the ſolid Content, Multiply the ſuperficial Content by the ſixth part of the Diameter, *viz.* $1145\frac{1}{3}$, and it brings forth $169881600314\frac{2}{3}$, which are ſo many ſolid Miles; or the ſolid Content in Miles

(of

(of 6120 feet in a Mile) of the whole Terreſtrial Globe, containing *Sea* and *Land*. And ſo much concerning the Meaſuring of the Earth in general: It follows that we Treat of *Surveying* and *Navigation* in particular? Only by the way take theſe two Corollaries.

1. Suppoſing the aforeſaid Diameter of the Earth 6872 $\frac{10}{21}$ Miles. If you imagine a hole through the Earth, and that a Milſtone ſhould be let fall down into this hole, and to move a Mile in a Minute of time, it would be more than two days and a half before it would come to the Center, and being there, would go no further, but as it were, hang in the Air.

2. Suppoſing the Circumference 21600 Miles, If a Man ſhould go every day 20 Miles, it would be three years wanting but a Fortnight before he could go once about the Earth.

And if a Bird ſhould fly round about it in two days, then muſt the motion be 450 Miles in an hour.

OF

OF SURVEYING.

CHAP. II.

Surveying in a ſtrict and limited ſenſe may be taken only for the Meaſuring of ſome piece or pareel of Land: But in a larger ſenſe, I take it here for the taking of all manner of Heights and Diſtances at Land, whether the ſame be Acceſſible, or Inacceſſible.

TAking of Heights being a Reſolving of a Triangle, ſtanding upwards or vertically upon his Horizontal Baſe, and taken by a Quadrant with Thread and Plummet: Taking of Diſtances is a reſolving of a Triangle (ſuppoſed to lye Horizontally Flat,) and taken by a Theodolite or ſuch like Inſtrument, which is no other but a Quadrant layd Flat

with

with an Index, and Sights inſtead of a Thread and Plummet: Not to exclude other Inſtruments, as the *Croſs-Staff*; for taking both Heights and Diſtances both at Sea and Land; but eſpecially at Sea: As alſo the *Plain Table*, an Inſtrument very much admired by many, of which you ſhall hear more hereafter.

As likewiſe I ſhall ſay ſomething of the Circumferentor, another Uſeful inſtrument for Surveying; but that which I do moſt admire and uſe, is the Quadrant for Heights, and the Theodolite for Diſtances, which is no more but a Circle, a Semicircle, or Quadrant layd Flat, divided into Degrees and Minutes with an Index and Sights, as any other Quadrant (properly ſo called) is held upright, and uſed with a Thread and Plummet.

Both theſe (that is) both the Quadrant and Theodolite may be notably ſupplyed by *Gunters* Sector, being ſet at the diſtance of a Quadrant (or 90 degrees) and having a looſe piece faſtned with two Mortices, and divided into Degrees for the quadrantal ſide, having alſo a Thread and Plummet faſtned in the Center, when you uſe it one way for a Quadrant, and a Pin for an Index and Sights to turn upon when you uſe it the other way for a Theodolite, being uſually each Leg a juſt foot in length

from

from the Center; the Quadrantal or looſe piece may be firſt divided into 90 degrees, and every degree into 6 parts, ſo that every one of the leſſer Diviſions are 10 Minutes, and theſe 10 may be ſuppoſed to be ſubdivided into other 10, which ſignifie ſingle Minutes, which is as much as may be for an Inſtrument of no larger Radius. When it is uſed as a Theodolite, there muſt be ſcrewed to it a ſocket of Braſs, whereby the whole Inſtrument may turn upon the top of a three-leg'd ſtaff (hooped alſo with Braſs, as there ſhall be occaſion) with a ſcrew on the ſide of the ſocket to faſten the Inſtrument upon the ſtaff in any poſition you deſire: There may be alſo (to make it compleat) a Card and-Needle ſcrewed upon the Center, and the Index which carries the ſights, being twice the Length of the Legs, (that is) 2 foot long, another of the ſame length without ſights may croſs it at Right Angles; ſo that making a perfect croſs, moving upon the Center as one part goes off the Quadrant, another comes on, which is as good as if it were a whole Circle. This may ſuffice for the deſcription of the Quadrant and Theodolite, I ſhall now ſhew you how to take Heights and Angles of Altitude by the Quadrant, and to take Diſtances or Angles of Poſition by the Theodolite: And to take both Heights

and

and Diſtances by the Croſs-ſtaff, whether the ſame be Acceſſible, or Inacceſſible.

SECT. 1.

To take any Height or Altitude, whoſe Foot or Baſis is acceſſible at one ſtation.

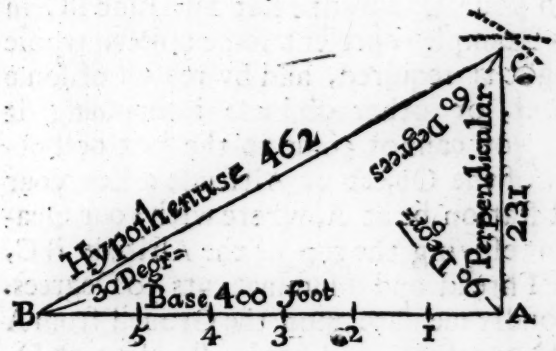
This is no more but the reſolving of a plain Right Angled Triangle. As ſuppoſe the perpendicular CA were a Tree, Tower, or Steeple, or any other thing whoſe Height is required. Firſt, ſtanding at any convenient diſtance from the foot of the Object to be meaſured as at B, and there looking through both the ſights of your Quadrant till you eſpie the top of the Altitude at C, obſerve what Degrees and Minutes are cut on the Limb of the Quadrant by the Thread, which ſuppoſe 30 degrees, as in the Example: Theſe 30 degrees are the quantity of the Angle CBA, (or more plainly to call it the Angle B.) Now becauſe that every right-lin'd Triangle contains 180 degrees of a Circle, and every right Angle contains 90 of them; the Angle A being a right Angle, the other two Angles, that is B and C muſt contain other 90, whereof B by the Quadrant appears to be 30: Therefore C muſt needs be 60 degrees, the complement of 30 to 90. And laſtly,

You

You muſt meaſure by a Yard, Chain, or a Two foot Rule, the Baſe of the Triangle, that is, the ſpace or diſtance betwixt B, the place of your ſtanding, and A the foot or bottom of the Altitude (which ſuppoſe to be 400 foot) As in the Example.

Now theſe three parts of the Triangle being given or known, that is, two Angles and a ſide, *viz.* the Angle B 30 degrees, and by conſequence the Angle C 60 degrees, and the length of the Baſe 400 Foot: You may by Trigonometry and the Rule of Proportion find any of the other three parts of the ſame Triangle, *viz.* the Angle A, the Hypothenuſe BC, or (that which is here required) the Height or Length of the Perpendicular CA. You will find that any Angle or ſide, (and conſequently this ſide the perpendicular here required) may be found ſeveral ways, but by theſe three waies eſpecially: Firſt, By Protraction, *pag.* 85, 86, 87. Secondly, By Arithmetical Operation, *pag.* 137, 138. But eſpecially by that general Rule in Trigonometry, *pag.* 141, that every Angle is proportional to its oppoſite ſide, and every ſide to its oppoſite Angle: Therefore as the Angle C 60 Degrees is to the known ſide BA 400; ſo is the Angle B 30 Degrees to its oppoſite ſide ſought; the Perpendicular CA, or the Height required, which is no more

more but performing the Rule of Three (or by these three Numbers given, to find a fourth;) either first by Logarithmes, as is taught *pag.* 41. Or, Secondly, By the Lines of Numbers, Sines and Tangents, *pag.* 41. Or lastly, by the Sector, *pag.* 150, 151. by placing the Numbers on this manner.



If 60 degrees give 400 Foot, what gives 30 degrees? The Answer by all the forementioned ways will be found to be 231 *ferè* the length of the perpendicular, or the Height required: But to avoid Repetition, I shall refer the Reader to the pages aforementioned for the manner of Operation.

SECT.

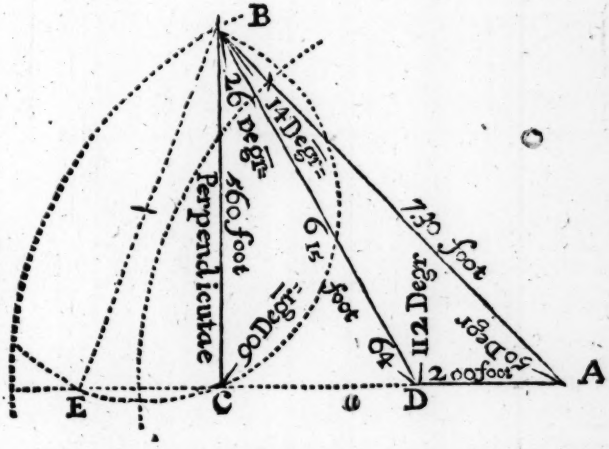
SECT. 2.

To take an Inacceſſible Altitude at Two Stations.

For the effecting hereof, you muſt take two obſervations at two ſeveral Stations with your Quadrant. Let the Line BC in this Example repreſent ſome object, whoſe Height is required, and by reaſon of ſome Water, or other obſtacle interpoſing it ſelf, you cannot come to the foot or bottom of the Object or Altitude: Let your firſt ſtation be at A, where with your Quadrant eſpying the top of the Altitude B C, the Thread and Plummet cuts 50 Degrees. Secondly meaſure upon the Ground from A to D 200 foot, and again ſtanding at D, and eſpying the top of B C, obſerve what Degrees are cut by the Thread upon the Limb or Edge of your Quadrant, which ſuppoſe 64 *d.* Then fall to Protracting, and firſt draw an Angle at A 50 Degrees, and meaſuring 200 foot to D, protract there at D another Angle of 64 degrees, and from B, (that is from the point where the two lines AB and BD do interſect or cut one another,) let fall a perpendicular, as is taught *pag.* 55. *Probl.* 3 and meaſure the Length of the ſaid perpendicular by the ſame ſcale

by

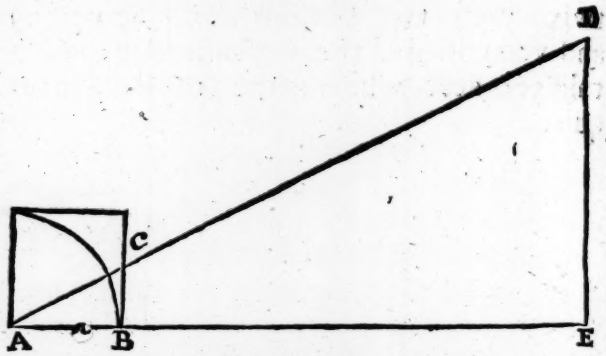
by which you laid down the distance betwixt your two stations (*viz.* 200 foot) and you will find the perpendicular to contain 560 Feet, which is the true Height required.



You may also do this by resolving the two Triangles by Logarithmes, or by the lines of Numbers, Sines, and Tangents, *&c.*

Note that in taking all manner of Altitudes, whether accessible, or inaccessible, you must alwaies add to the height found, the height of your Instrument from the ground.

Heights may be also taken by an Astrolabe: As also by a Quadrant, which represents the Tangents of a Circle (as in the following



following Example,) with an Index and Sights; for as the Radius A B is to the Tangent BC, ſo is AE the diſtance to ED the Altitude.

SECT. 3.

By the Height of the Sun, and length of the ſhadow, to find the Height of any Tree, Houſe, Steeple, or the like.

Firſt, Take the Suns Height by a Quadrant, which ſuppoſe 37 Degrees, and the length of the ſhadow of the ſaid Tree, (which ſuppoſe 40 Feet) Then ſay as Radius to the length of the ſhadow of the Tree 40 Feet, ſo is the Tangent of the Suns height 37 Degrees to the height of the Tree deſired, *viz.* $30\frac{14}{100}$ feet.

Note that when the Sun is 45 degrees high

high, then the ſhadow is equal to the height; at 26 *d.* 56 *m.* the length of the ſhadow is double to the height; at 18 *d.* 43*m.* it is 3 times; at 14*d.* 4 *m.* it is 4 times the Height, and at 11 *d.* 31 *m.* the ſhadow is 5 times the Height of any Altitude. Or, you may know the Height of any thing by a walking-ſtaff, or a Yard, or a two foot Rule by the ſhadow, thus.

Set the ſtaff upright, and meaſure how often the ſhadow of the ſtaff is longer or ſhorter than the ſtaff, ſo in proportion is the ſhadow of the Tree to the height of the Tree it ſelf. The like may be done by a Maſons Square ſet upright.

SECT. 4.

To take the Height of any thing acceſſible by 2 ſtraws or ſticks laid croſs one another.

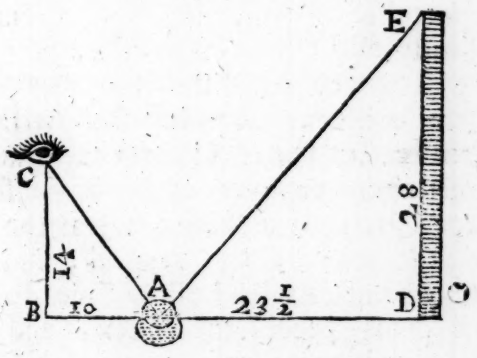
Take two ſtraws or ſticks of equal length, and faſten them perpendicularly croſs one another juſt in the middle of both, and go backwards or forwards till the bottom and top of the Altitude be even with the ends of the Croſs-ſtick, laying your Eye to the end of the other ſtick, and then is the height exactly as much as the diſtance meaſured to the Foot of the Altitude, adding to the Height found the Height of your Eye from the ground.

SECT.

SECT. 5.

To take an Altitude by a Bole of Water, or an ordinary Looking-glaſs.

Place on the ground a Bole of Water, which done, erect your body ſtrait up, and go back in a right line from the building or other Altitude, till you eſpie in the Center or Middle of the Water the very top of it; which done, obſerve the place of your ſtanding, and meaſure the height of your Eye from the ground together with the diſtance of your ſtanding from the Water; and the diſtance of the Water from the Baſe or Foot of the Altitude: Then ſay as the diſtance from your ſtanding to the water is to the Height of your Eye; ſo is the diſtance from the Water to the Foot of the Altitude, to the Height of the Altitude required. The ſame may be done by a Glaſs, if the Glaſs be placed Horizontal, or Flat upon the ground, and the Eye ſee the top of the Tower, *&c.* in the Angle or edge of the Glaſs, and in the Line of Reflection: I ſhall give you only one Example for both: Suppoſe the Bole of Water or the Glaſs be A, which laid flat upon the Ground, or Horizontal; go backwards till you eſpie the top of the Altitude E, then meaſure how many



many parts is from your ſtanding at B to A, and alſo from B to your Eye at C, likewiſe the diſtance from A to D, and then ſay as the diſtance B A 10 is to the height of your Eye from the Ground CB 14, ſo is the diſtance from A to D the foot of the Altitude 23½ to the Altitude required, which you will find 28 of the ſame parts.

CHAP. III.

THus much may ſuffice for taking of Heights, in the next place we come to ſhew How to take *Diſtances*, and that (as I told you before) is done the beſt by the Theodolite, which is the moſt excellent

Inſtrument for that uſe (eſpecially as I have contrived it upon the Sector) and alſo the moſt portable for conveniency of carriage; I ſhall only add this contrivance, that inſtead of a three leg'd ſtaff you may have only an ordinary walking-ſtaff with an Hoop inſtead of the Head for the ſocket of the Inſtrument to move upon, and inſtead of three legs, only a Braſs cheap at the end to prick it down in the ground; and the ſtaff may be made with hollow Caſes to put in the croſs peece, the Index, and the Sights, and if you pleaſe, a pair of Compaſſes, whereby it becomes a general Inſtrument both for obſervation and Calculation; the Sector you carry in your Pocket, and the Staff in your hand, appearing no other but an ordinary walking Staff, which for neatneſs and portability exceed any other contrivance I meet with.

SECT. 1.

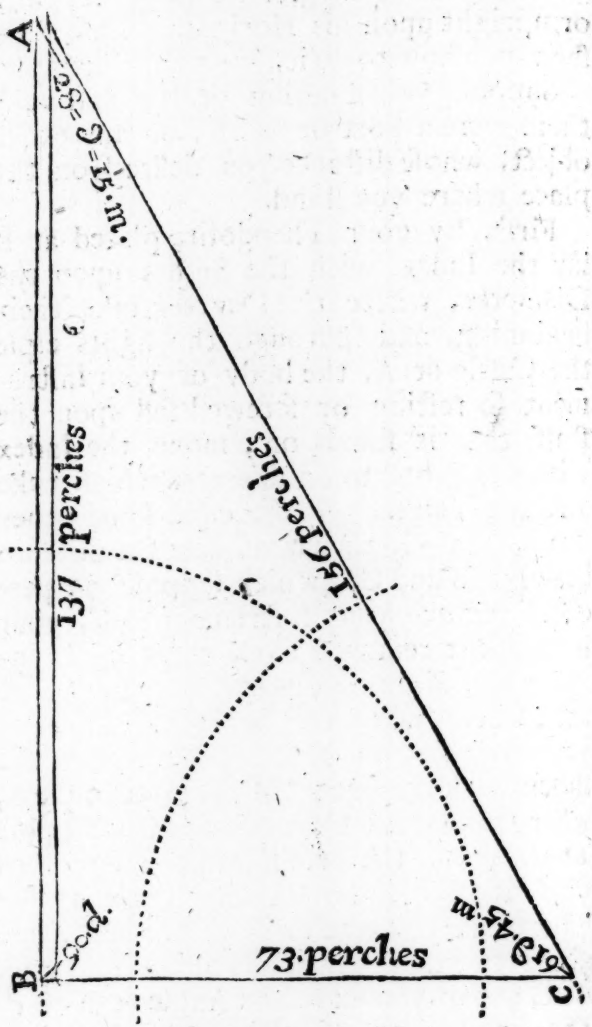
But to come to Practice, and by the Theodolite to find the Length or Diſtance of any place from you, how far ſoever it be.

This (as was formerly hinted) is no more but the reſolving of a Triangle ſuppoſed to lye flat or Horizontally upon the Ground; As the taking of Heights is the reſol-

reſolving of a Triangle, ſtanding vertically or upright upon his Horizontal Baſe: To ſhew you how to do it, take this Example.

Suppoſe you ſtanding at B, and at A there were a Fort or Caſtle, or any other object, whoſe diſtance you deſire from the place where you ſtand.

Firſt, By your Theodolite placed at B lay the Index with the Sights upon the Diameter, where the Degrees take their beginning, and thorough the ſights eſpie the Caſtle at A, the body of your Inſtrument ſo reſting or ſcrewed faſt upon the ſtaff, that it ſtands on, move the Index with the ſights to 90 degrees, which make a Right Angle. There eſpie ſome other mark, as at C, then meaſuring the diſtance betwixt B and C, (which ſuppoſe 73 perches) remove your Inſtrument to C, your Inſtrument removed to C, there with the Index and Sights upon the Diameter where the Degrees take their beginning, Eſpy out your firſt ſtation at B, then turning the Index with the ſights till you again eſpie A, where note what Degrees are cut; ſuppoſe 61 *d.* 45 *m.* the complement whereof to 90 *d.* is 28 *d.* 15 *m.* for the Angle at A: Therefore you have two Angles and a ſide given to find another ſide of the Triangle, ABC; for you have the Angle at A 28 *d.* 15 *m.* and its oppoſite ſide BC. 73 perches,


A
137 perches
156 perches
73 perches
B
C

and again you have the Angle at C 61 *d.* 45 *m*, which holds the ſame proportion to its oppoſite ſide BA, which is the diſtance required.

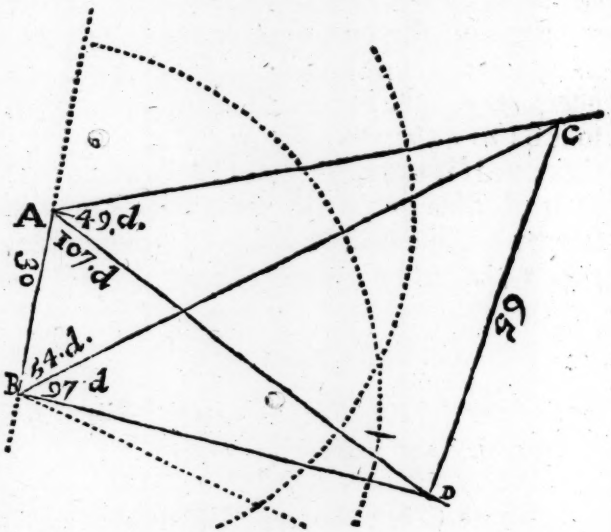
Which ſide or diſtance you may find by Reſolving the Triangle either by Protraction, as in *pag.* 132, 133. or by Arithmetick working by Logarithmes, as *pag.* 31. or by the Lines of Numbers, Sines and Tangents, *pag.* 41. for as 28 *d.* 15 *m*, is to 73, ſo is 61 *d.* 45 *m.* to the ſide B A 156 perches: and (if the other ſide C A be required) ſo is Radius or 90 degrees to its oppoſite ſide by the general Rule in Trigonometry, *pag.* 141. being 156 perches.

SECT. 2.

To find the diſtance betwixt any two Forts, and yet come near neither of them.

Suppoſe C one Fort, and D another, and you are to find the diſtance betwixt them, and come nearer neither of them than A and B. Firſt place your Inſtrument at A, and laying your Index upon the Diameter where the Degrees take their beginning, through the ſights eſpie the Fort at C; the Inſtrument ſo reſting, move the Index till through the ſights you eſpie D, which will be when the Index cuts upon the Limb 49

Degrees, which is the quantity of the Angle CAD; Then moving your Index till through the Sights you eſpie your ſecond ſtation at B, then you will find the Index to cut 107 Degrees, which is the quantity of the Angle CAB: Then meaſure the di-



ſtance betwixt A and B your firſt ſtation and ſecond, which ſuppoſe 30; then removing your Inſtrument to B, lay your Index with the ſights upon the Diameter where the Degrees take their beginning, and move the body of the Inſtrument upon the ſtaff, till through the ſights you eſpie the place of your firſt ſtation A, there ſcrew your

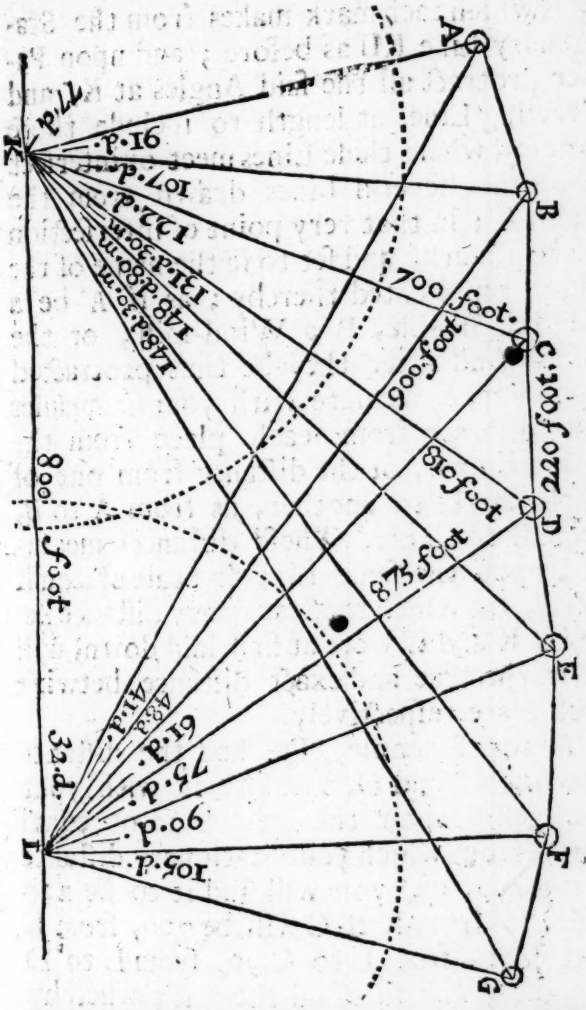
your Inſtrument faſt, and move the Index till through the ſights you eſpie the firſt Fort again at C, and obſerve how many Degrees are cut by the Index, which you will find 54 degres, or the Angle ABC: Laſtly move your Index, till through the ſights you eſpie the ſecond Fort at D, and there obſerve what degrees are cut by the index, which you will find 97 degrees, or the Angle ABD. Note theſe ſeveral obſervations as you take them, and then upon Paper protract the ſame: As Firſt, Draw your Stationary Line AB, and upon the point A protract the Angles 49 and 107 *d.* Likewiſe upon B protract the Angles 54 and 97 *d.* and laſtly draw the line CD from the points of Interſection at C and D, and that meaſured upon the ſame Scale whence your Stationary diſtance was taken, will ſhew you the true diſtance betwixt the Forts C and D, which was the thing required.

SECT. 3.

How to take the Diſtance of divers places one from another according to their true Scituation in Plano, ſtanding at ſome convenient diſtance.

Make choice of two convenient Stations (taken as large as you can, whereby your

work will be the truer,) from which ſtations you may plainly diſcern all the marks or places whoſe Diſtances you would take: As ſuppoſe K and L to be your two Stations; Firſt then, place your Inſtrument at L, and turn the whole Inſtrument about, till through the ſights on the Index lying upon the Diameter, (that is, where the Degrees take beginning) you eſpie your ſecond ſtation at K, which found, faſten your Inſtrument: Then moving your Index with the ſights, till you eſpie the mark at A, noting how many degrees are cut by the Index to the end you may protract the ſame when you have done your obſervation, and after the ſame manner move the Index, till through the ſights you eſpie all the reſt of the marks one after another, as BCDEF, noting down the ſeveral Angles from the Stationary Line KL. Then removing your Inſtrument from L to K, meaſure the Stationary diſtance betwixt L and K, which ſuppoſe 800 foot, as in the Example, when you come to protract, prick out this diſtance from a Line of equal parts upon the line K L, and placing your Inſtrument at K, lay the Index upon the Diameter, and turn the whole Inſtrument about till through the back ſights you eſpie your firſt ſtation L, and there ſcrew it faſt; then moving the Index, direct your ſight



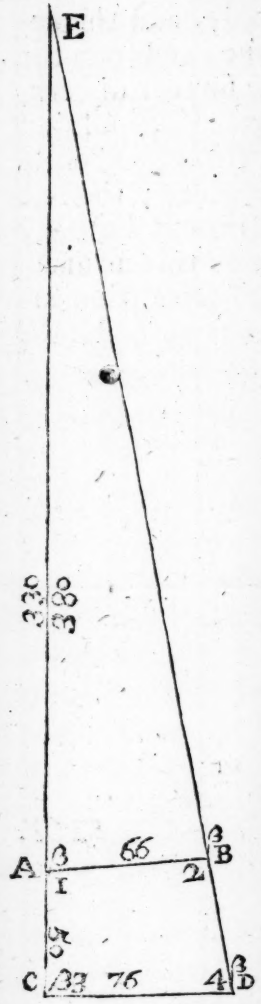
so as to espie the several marks at ABCDE FG, observing and noting the several An-

gles which each mark makes from the Stationary Line KL as before; and upon Paper protract all the ſaid Angles at K, and drawing Lines at length to include thoſe Angles, where theſe Lines meet or interſect the other ſeveral Lines drawn from the point L; in that very point of interſection make a mark, and ſet to it the name of the thing repreſented thereby; as if A be a Church-Steeple, B a Wind-mill, or the like: And after they be thus protracted upon Paper, meaſure with your Compaſſes the diſtance from each place from the point K or L, or the diſtance from one of theſe marks to another, as from A to B, from B to C, *&c.* Theſe diſtances meaſured upon the ſame Line or Scale of equal parts, by which the Stationary diſtance betwixt K and L were at firſt laid down, will ſhew the true and exact diſtance betwixt each place reſpectively.

As for Example, To find the diſtance betwixt C and D, meaſure it with your Compaſſes upon the ſame Line of equal parts, from which your Stationary diſtance was taken, and you will find it to be 220 foot, ſo from K to C will be 700, from K to D 810, from L to C 90, from L to D 875 foot, and ſo of all the reſt; whereby you may perceive this is the ſame with what is taught *pag.* 197; only that is taking the diſtance

diſtance betwixt 2 places only, and this betwixt many at the ſame time, and upon the ſame ground; and you may further diſcern, that theſe are no more but the Reſolving of Triangles by Protraction, wherein two Angles and a Side are given; the reſt of the Sides and Angles (if need be) may be found as by protraction on this manner, ſo likewiſe by Arithmetical Operation alſo, which I leave on purpoſe to exerciſe the Readers Ingenuity, and paſs on.

SECT.



SECT. 4.

To take a long Distance without the help of any Instrument, unless it be only such an one that will direct you readily to set out a Right Angle, as any ordinary Carpenters Square, or the like.

Observe the Figure, and let C be your standing place, and let E be the mark afar off, whose distance from C you would know.

First, Move in a right Line between C and E to A any number of yards or perches, as suppose 50, and set up a Staff at A, then move in a perpendicular Line to C E from A to B, making a right Angle at A any distance, suppose 66, and set up another staff at B, then come back again to C, and remove

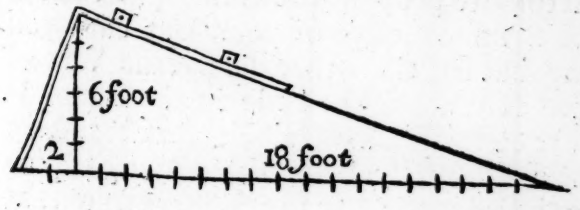
move in a perpendicular Line to CE, till you ſee the mark ſet up at B, and the point E in a Right Line, and ſet up another Staff at that place at D, getting the exact Diſtance thereof from C, which ſuppoſe 76; then ſubſtract the meaſured Diſtance AB 66 from the meaſured Diſtance CD 76, and note the Remainder, which is 10. Then by the Line of Numbers, or by the Rule of Proportion.

As the Difference between AB and BC 10 is to the diſtance between A and C 50, ſo is the meaſured Diſtance CD 76 to the Diſtance between C and E 380. Or, So is the meaſured Diſtance AB 66 to the Diſtance AE 330.

SECT. 5.

To take a Diſtance by a Carpenters, or Maſons Square.

Take a Square with two Sights on one ſide, faſten it to the top of a Staff, which ſuppoſe 6 foot high, move the Square, ſo



that

that through the Sights you eſpie the thing which you deſire to know how far it is diſtant from you: Then obſerve the diſtance betwixt the Staff (faſtned to the other ſide of the Square where the ſights are not) and the 6 foot ſtaff which ſuppoſe 2 foot; Then obſerve as often as that diſtance is contained in the length of the ſtaff, ſo many times is the length of the ſtaff contained in the Diſtance required. As how many times 2 in 6, that is 3 times. Then ſay, 3 times 6 the length of the Staff is 18, the diſtance required. As in the Example.

SECT. 6.

To Meaſure an Inacceſſible Diſtance, as the Breadth of a River with the help of ones Hat only.

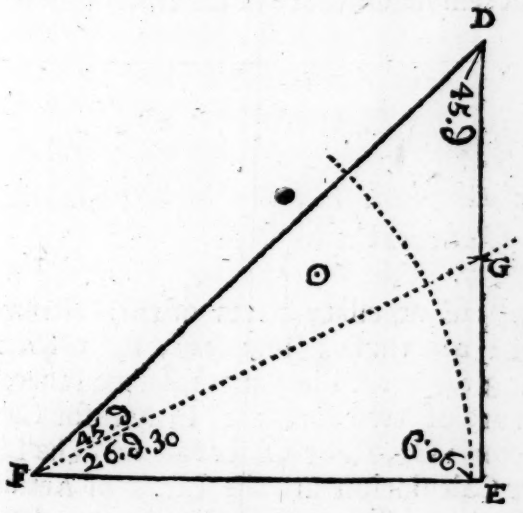
Having your Hat upon your Head, come near to the Bank of the River, and holding your head upright (which may be done by putting a ſmall ſtick to ſome one of your buttons to prop up the Chin) pluck down the Brim or edge of your Hat until you may but ſee the other ſide of the Water, then turn about the body in the ſame poſture towards ſome plain, and mark where the ſight by the brim of the Hat glanceth on the Ground: For the Diſtance from that place

place to your ſtanding is the Breadth of the River required.

SECT. 7.

How to take the Breadth of a River exactly.

Place your Theodolite, (or any other Inſtrument whereby you can take Angles;) at E obſerve ſome mark on the other ſide the River, as at D, and obſerve an Angle



of 90 Degrees betwixt D and F, which may be either on the Right Hand, or on the left, ſo far in the Line of 90 Degrees, till you

you eſpie the mark D doth juſtly make an Angle of 45 Degrees with the mark E, and this will be when you come to F; then meaſure carefully the Diſtance E F, and that ſhall be exactly equal to the Breadth of the River DE: An Angle of 26 *d.* 30 *m.* makes the Diſtance double to the Breadth, *&c.* and ſo by other Angles, but the beſt and ſureſt is 45 *d.* if you can uſe no other: So likewiſe by the ſame Rule may be taken the Diſtance of any Cape or Iſland from you at Sea, or any other Acceſſible Height, or Inacceſſible Diſtance at Land.

CHAP. IV.

To take both Heights and Diſtances by Gunters *Croſs-Staff.*

THE neceſſary parts of this Inſtrument are theſe, The *Staff*, the *Croſs*, and the 3 *Sights*. The Lines inſcribed thereupon are of two ſorts: 1. Either for Calculation: Or, 2. For Obſervation: The Lines for Calculation are the Lines of Artificial Numbers, Sines, and Tangents, whereof we have ſpoken largely, The Meridian Line for drawing of Sea-Charts; As alſo the Lines of verſed Sines, Chords, *&c.* which

which I ſhall ſay nothing of in this place. The ſecond ſort of Lines inſcribed upon the Inſtrument are for Obſervation: Firſt, A Line of Tangents for Obſervation of Angles. Secondly, A Line of equal parts not only for taking Angles, but alſo for taking perpendicular Heights and Diſtances. The Tangent Line on the Staff may be known by the double Numbers ſet on both ſides of the Line, beginning at one ſide at 20, and ending at 90, and on the other ſide at 40, and ending at 180. On the Croſs there are two Tangent Lines.

1. A Tangent Line of 36 *d.* 3 *m.* numbred by 5, 10, 15, to 35, the midſt whereof is at 20 *d.* and therefore I call it the Tangent of 20, and this hath reſpect unto 20 *d.* in the Tangent on the Staff.

2. A Tangent Line of 49 *d.* 6 *m.* numbred by 5, 10, 15, to 45, the midſt whereof is at 30 *d.* and hath Reſpect to 30 *d.* in the Tangent on the Staff, and therefore called the Tangent of 30. The Uſe of theſe follows.

SECT. 1.

To find an Angle by the Tangent on the Staff.

Let the middle Sight be alwaies ſet to the

the middle of the Croſs, noted with 20 and 30, and then the Croſs drawn nearer the Eye, until the marks may be ſeen cloſe within the ſights: For if the Eye at A (the end of the Staff) which is noted with 90 and 180, beholding the marks K and N between the two firſt ſights C and B, or the marks K and P between the two outward ſights, the Croſs being drawn down to H, ſhall ſtand at 30 and 60 in the Tangent of the Staff: it ſheweth the Angle KAN is 30 *d.* the Angle KAP 60 *d.* the one double to the other, which is the reaſon of the double Numbers on this Line on the Staff; and this way will ſerve for any Angle from 20 towards 90 *d.* or from 40 to 180 *d.* but if the Angle be leſs than 20 *d.* we muſt then uſe the Tangents on the Croſs.

SECT. 2.

To find an Angle by the Tangent of 20 *on the Croſs.*

Set 20 unto 20, that is, the middle ſight to the midſt of the Croſs at the end of the Staff noted with 20, ſo the Eye at A beholding the marks L and N cloſe between the two firſt ſights C and B, ſhall ſee them in an Angle of 20 degrees: If the marks ſhall be nearer together as are M and N, then

then draw the Croſs in from C to E: If they be further aſunder, as are K and N, then draw out the Croſs from C to F, ſo the Quantity of the Angle ſhall be ſtill found in the Croſs in the Tangent of 20 *d.* at the end of the Staff; and this will ſerve for any Angle under 20, and ſo to 35 Degrees.

SECT. 3.

To find an Angle by the Tangent of 30 *upon the Croſs.*

This Tangent of 30 is here put the rather, that the end of the Staff reſting at the Eye, the hand may more eaſily remove the Croſs: For it ſuppoſeth the Radius to be no longer than AH, which is from the Eye at the end of the Staff to 30 *d.* about 22 inches and 7 parts, wherefore here ſet the middle ſight to 30 *d.* on the Staff, and then either draw the Croſs in or out until the marks be ſeen between the two firſt ſights: So the quantity of the Angle will be found in the Tangent of 30, which is here repreſented by the Line GH, and this will ſerve for any Angle from 0 *d.* towards 48 degrees.

SECT.

SECT. 4.

To take the Altitude of the Sun by Thread and Plummet.

Let the middle ſight be ſet to the midſt of the Croſs, and to that end of the Staff which is noted with 90 and 180; then having a Thread and Plummet at the beginning of the Croſs, and turning the Croſs upwards, and the Staff towards the Sun, the Thread will fall on the complement of the Altitude above the Horizon, and this may be applyed to other uſes alſo. Thus much for the Line of Tangents.

The other Line for Obſervation, whereby you may find both Angles and Sines is the Line of equal parts, which in *Gunters* upon the Staff is 36 inches, each inch divided into Tens, and each Ten into Halfs; and to anſwer it upon the Croſs, is 26$\frac{1}{2}$ inches between the two outward ſights.

SECT. 5.

To take Angles by theſe Lines of equal parts.

If the Angles be obſerved between the two firſt ſights, there will be ſuch proportion between the parts of the Staff, and the

the parts of the Croſs, as between Radius, and the Tangent of the Angle; as if the parts intercepted on the Staff be 20, the parts of the Croſs 9: Then by Proportion, as 20 is to 9 upon the Line of Numbers, ſo is the Radius to the Tangent of 24 *d.* 14 *m.* upon the Line of Tangents: But if the Angle be obſerved between the two outward Sights, the parts being 20 and 9, as before, the Angle will be double to the former, *viz.* 48 *d.* 28 *m.*

SECT. 6.

To take not only Angles, but Sides alſo by this Line of Equal Parts, which is, To take Heights and Diſtances.

And Firſt, For Heights.

In taking of Heights, you are to hold the Croſs upwards, and for Diſtances, to hold the Croſs ſidewaies, and the Eye placed at the beginning of the Staff, eſpying the marks by the inward ſides of the Sights, there will be ſuch proportion betwixt the Diſtance and the Height as is between the parts intercepted on the Staff and Croſs: The Ground whereof you will find in *pag.* 189.

PROBL.

PROBL. 1.

Having the Diſtance from your ſtanding to the foot of the Altitude given, To find the Height.

As the Segment of the Staff is to the Segment on the Croſs, ſo is the Diſtance to the Height. As if the Diſtance AB being meaſured, or known to be 256 feet, it were required to find the Height BC: Firſt, I place the middle ſight at 24 inches (or parts,) then holding the Staff level with the Diſtance, I raiſe the Croſs parallel to the Height or Top of the Altitude; ſo as my Eye may ſee from A, (the beginning of inches, or parts upon the Staff) by the ſight E unto the mark C; which being done, I find 18 inches intercepted on the Croſs between the ſights at E and D: Therefore I ſay, the Height BC is 192 feet; for as 24 is to 18, ſo is 256 to 192: Or if the obſervation were to be made before the Diſtance were meaſured, I would ſet the middle ſight to 12, 16, 20, 24, or ſome ſuch Number as might beſt be divided into ſeveral parts, and then work by Proportion: As in the former Example, Suppoſe I ſet the Staff to 24, and the Croſs to 18, it ſhews the Height is $\frac{3}{4}$ of the diſtance, therefore the Diſtance being 256, the Height muſt be 192 as before.

PROBL.

PROBL. 2.

Having Part of the Height given, To find the whole Height.

As if the Height from G to C were known to be 48, and it were required to find the whole height BC. Firſt, put the third ſight, or ſome other ſight upon the Croſs between the Eye and the mark G. Then, As the Difference between the ſights E and F 45, is to the whole Segment on the Croſs E D 180, ſo is the Part of the Height given G C 48 to the whole Height BC 192.

PROBL. 3.

To find an Inacceſſible Height at 2 Stations, by knowing only the Stationary Diſtance.

As the Difference of Segments on the Staff is to the Stationary Diſtance, ſo is the Diſtance on the Croſs to the Height required: As ſuppoſe the firſt Station at H, the Segment on the Croſs E D were 180, and the Segment of the Staff HD 300. Then coming 64 Feet nearer to B in a direct Line towards the Height unto a ſecond ſtation at A, and making another obſervation: Suppoſe the Segment of the

Croſs

Croſs ED were ſtill 180 as before, and the Segment of the Staff AD 240. Take 240 out of 300, the Difference of Segments will be 60: Then ſay,

As 60 the Difference of Segments to 64 the Stationary Diſtance (or Difference of Stations) ſo is DE 180 the Segment of the Croſs to the Height required, *viz.* BG 192.

PROBL. 4.

Having the Height of any thing given, to find the Diſtance.

Secondly, For finding Diſtances.

As the Segment of the Croſs (or the parts upon the Croſs which is all one) is to the Segment of the Staff, (or the parts upon the Staff,) ſo is the Height given to the Diſtance required. For Example, as the Segment ED 18 on the Croſs is to DA 24 on the Staff, ſo is CB 192 the Height to 256 the Diſtance.

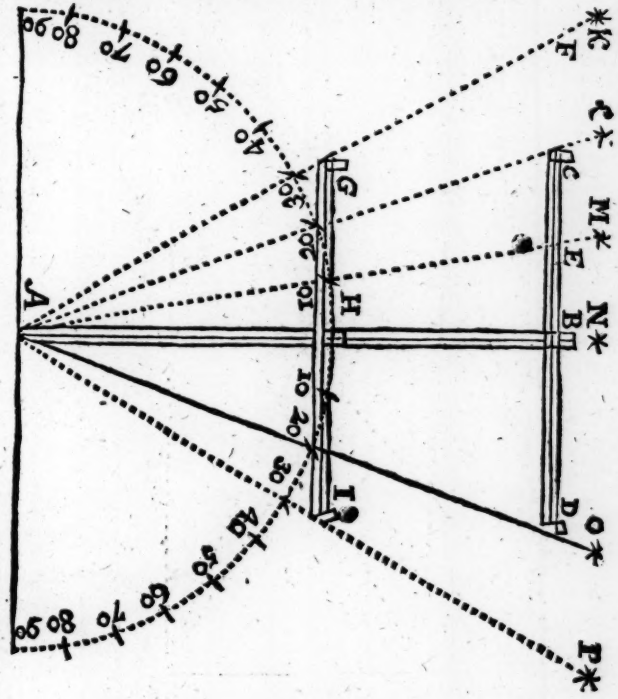
PROBL. 5.

Having Part of the Height given, To find the Diſtance.

As the Difference between the Sights

EF 45

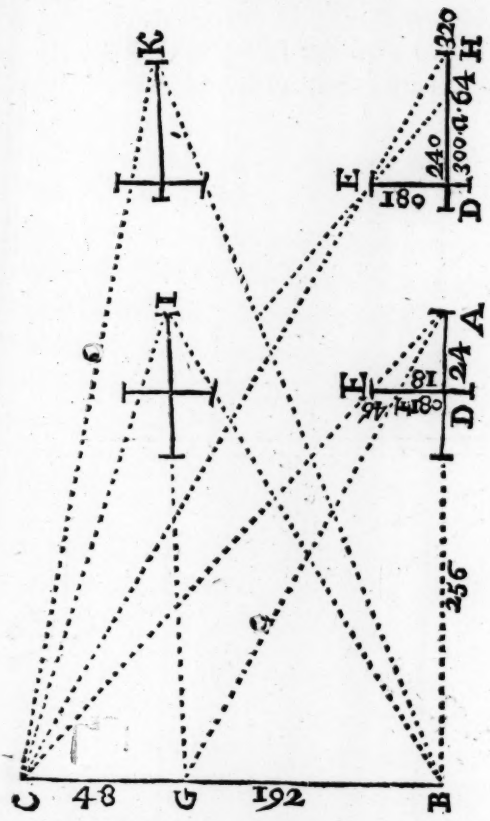
EF 45 is to the whole Segment of the Staff DA 24, ſo part of the Height GC 48 is to the diſtance required, *viz.* AB 256.



PROBL. 6.

To find a Diſtance at two Stations, by knowing only the Stationary Diſtance.

As the Difference of Segments on the Staff is to the Stationary Diſtance, ſo is



the whole Segment of the Staff to the distance required. The Segment of the Croſs being as before 180 The Segment of the Staff at the firſt Station 240, at the ſecond 300. The difference between theſe 60. The diſtance between the two Stations 64. Then ſay,

As

As 60 the difference of Segments is to 64 the Stationary diſtance, ſo is the Segment at the firſt ſtation 240 to the diſtance AB the firſt ſtation 256. And ſo is 300 the Segment at the ſecond Station, to 320, the diſtance HB the ſecond Station.

In taking a Breadth it is not material whether the two Stations be choſen at one end of the Breadth propoſed, or without it, or within it; If the Lines between the Stations be perpendicular unto the Breadth; as may appear if inſtead of the Stations at A and H, we make choice of the like Stations at I and K. To conclude, In all theſe there is regard to be had to the parallax of the Eye, and his Height above the Horizon; to the Semidiameter of the Sun, his Parallax and Refraction: As alſo the Parallax and Refraction of the Moon and Stars, if you will be ſo very exact; which you may ſee in this following Table.

The Table of these Parallaxes according to the Observations of *Tycho Brahe.*

The Table of Refractions of the Sun, Moon, and Stars.

Altitudes.	Sun Min.	Moon min.	Stars min.	Altitudes	Sun min.	Moon min.
0	34	33	30	18	6	6
1	26	25	21	19	5	6
2	20	20	15	20	4	5
3	17	17	12	21	4	5
4	15	15	11	22	3	4
5	14	14	10	23	3	4
6	13	13	09	24	3	4
7	12	13	8	25	2	3
8	11	12	7	26	2	3
9	10	11	6	27	2	3
10	10	11	5	28	2	2
11	9	10	5	29	2	2
12	9	09	4	30	1	2
13	8	09	4	31	1	2
14	8	8	3	32	1	1
15	7	8	3	33	1	1
16	7	7	2	34	1	1
17	6	7	2	35	1	1

The Refraction of the Sun, Moon, and Stars causeth them to appear higher above the Horizon than they really are, Therefore Substract the Refraction from the Altitude observed, that the true Altitude may be had.

The

The cauſe of theſe Refractions or Parallaxes is the Atmoſphear, or the groſs Vapours and Thickneſs of the Air near the Horizon, or the Face of the Earth; which to demonſtrate by an eaſie experiment, ſet down an empty Baſon on a Stool, laying a ſhilling in the bottom of it; and go ſo much backwards, till you bring the edge of the Baſon and the Shilling in a right Line with your Eye, and then let another fill the Baſon with fair Water, and you may go a pretty way further backwards, and ſtill ſee the Shilling and the edge of the Baſon in a right Line, the Refraction of the Water being the Cauſe thereof; and from this Reaſon we conclude that the Sun ſeems to appear above the Horizon when he is really ſet; and that other times he ſeems higher than he really is, as alſo the Moon and Stars.

CHAP. V.

HAVing ſhewed you ſeveral ways to take Heights and Diſtances in general, I come now to ſpeak of *Surveying*, as it is taken in a ſtrict and particular ſenſe for the Meaſuring of Land according to the Diverſity of Incloſures: Now this Meaſuring of

Land is no more but finding the Area or Content of a Superficies, and conſiſts in theſe two Particulars. Firſt, Plotting of Ground, or taking the Figure of it: Secondly, the Meaſuring of that Plot or Figure ſo taken, or taking the Superficial Content thereof.

1. The former of theſe, *viz.* The plotting of ground is excellently performed: Firſt, By the Plain Table. Secondly, By the Theodolite: Or, Thirdly, By the Chain only, and Scale and Compaſs.

2. The latter, *viz.* The Meaſuring of a Plot is no more but the Reducing of it to one or more of the Three Regular Geometrical Figures before-mentioned, *viz.* either to a *Circle*, a *Square*, or a *Triangle*, (eſpecially the laſt of theſe) and meaſuring it by their Rules, ſeeing no Plot or Figure can be ſo Irregular, but may be Reduced to Triangles, (if not to either of the former) and ſo may be meaſured by *Trigonometry*.

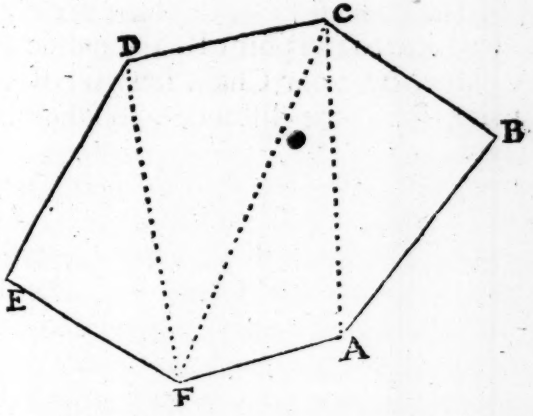
SECT. 1.

Firſt then, To take a Plot by the Plain Table.

A Plain Table is no more but a Board, or rather three pieces put together, making a long Square or Parallelogram, ſufficient to hold a ſheet of Paper, with Ledges

to

to keep the Table together, and the Paper faſt, with a looſe Index with Sights to lye upon the Table: The uſe is as follows: Firſt, Placing your Inſtrument upon the Staff, faſten it by the ſcrew in the ſocket, which it hath common with a Theodolite, and ſtanding at any Corner of the Incloſure, whoſe Plot you would take as at A, lay the Index with the ſights upon the Paper, and thorough the ſights eſpie ſuch a mark upon the Hedge or Side of a Field or Incloſure as far as the Hedge goes in a ſtreight Line as at B; and by the Fiducial



edge of the Index, draw a Line what length you pleaſe from A towards B: Then removing your Inſtrument meaſure with your Chain the diſtance betwixt A and B, and

from a Scale of equal parts prick down in Perches the ſaid Diſtance from A to B upon the Line drawn; then placing your Inſtrument at B, lay the Index upon the Line A B, and turn the whole Inſtrument till through the ſights you eſpie A your firſt ſtation; there ſcrew the Inſtrument faſt upon the Staff, obſerving that A lay even with the Line drawn, then moving the Index to the Point B, keep one end thereof to that point; and move the other till through the ſights you eſpie a third mark at C, (which again is as far as the Hedge or Side of the incloſure goes in a right Line:) And by the fiducial Edge of the Index draw another Line to the point B, and meaſuring the diſtance by your Chain betwixt B and C, prick the ſame diſtance upon the Line BC from the ſame ſcale of equal parts, and remove your Inſtrument to C, and ſo from Point to Point, till you have gone round the incloſure: So ſhall you have a perfect Plot of the whole Field upon your Paper; and having ſo, it is eaſily reduced to Triangles, as is taught *pag.* 102. and meaſured by its Rule, *pag.* 107. whereunto I refer the Reader for brevity.

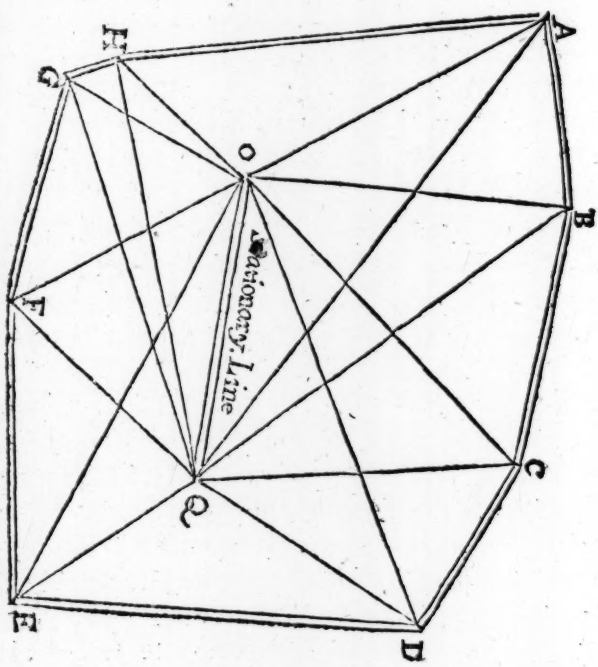
SECT.

SECT. 2.

Secondly, To take a Plot by the Theodolite.

The Deſcription of a Theodolite you have *pag.* 183, and 194. The uſe of it in taking diſtances in general, you have alſo *pag.* 194, 195, 196. 197, 198; and as for meaſuring of Land thereby, it is no more but the particular application of thoſe generals upon this particular occaſion: As for taking the former Plot in *pag.* 223 by this Inſtrument, placing your Inſtrument in any Corner of the Field, or in any point of any of the ſides thereof; your buſineſs is to take the diſtance betwixt one point and another till you have gone round the Field, as is taught 194, 195, and 196: Therefore I ſhall not here uſe Repetition: But becauſe there is one ſpecial Example in *Leyburn*, how by the Theodolite to Plot a Field at two Stations taken in the middle thereof; from either of which all the Angles in the Field may be ſeen with meaſuring the Stationary Line only, being an excellent, pleaſant, and ingenious way, I ſhall here give it you: And for brevity ſake I ſhall acquaint the Reader, that it is the very ſame as is ſhewed *pag.* 200, and 201, K L being

ſuppoſed the Stationary Line in the middle, and the whole Figure in *pag.* 120, being ſuppoſed to be only one half of the Plot, and Lines drawn from one point to another round the whole, as is there done; by which means you will have the Periphery of the whole Plot; but to give you an entire Example or Figure more, referring you to the former place for Direction, obſerve this following.



SECT.

SECT. 3.

Thirdly, To take the Plot of a Field without any other Inſtrument but the Chain only, and by your Scale to Protract the ſame upon Paper.

Let ABCDEF be a Field to be plotted, Firſt, Meaſure every ſide thereof with your Chain beginning at A, ſo ſhall you find, The ſide AB contains 2 Chains 50 Links

BC	1	48
CD	2	6
DE	4	75
EF	2	15
FA	2	52

Note that theſe are meaſured by Mr. *Gunters* Chain (which I do moſt commend, and which contains 4 Poles, each Pole 25 Links, or 100 Links in the whole Chain, each Link contains 7,92 inches, in all 792 Inches, 66 Foot, or 22 Yards.)

Now after that you have taken or meaſured theſe ſeveral ſides by this Chain, to Protract the ſame upon Paper, obſerve, That the Field contains 6 ſides, and therefore may be Reduced into two Trapezia's, or into 4 Triangles. A Trapezia is any Figure of 4 ſides, whoſe ſides and Angles are all unequal, ſo that this field may be reduced

ced into the Trapezium ABCF, and FCED by a Line drawn from the Angle F, to the Angle C, which being measured by your Chain, you will find the Length of it to be 2 Chains 89 Links. Then again, if you measure with your Chain from C to A, which will contain 2 Chains, 70 Links, and from C to E, which contains 3 Chains, 60 Links; you will find the former two Trapezia's divided each into two Triangles, *viz.* the Trapezium ABCF into the Triangles ABC and ACF: And the other Trapezium FCED into the Triangles FEC and CDE; in every which Triangle you have all the sides given, by means whereof you may draw upon Paper the exact Figure of your Field by what Scale you please, (according to the Rule given, *pag.* 85.) and as it here follows.

First, Taking into your consideration the Scituation of your Field, draw upon your Paper the Line A C, containing 2 Chains, 70 Links of any scale: Then because the side AB contains 2 Chains, 5 Links, open your Compasses to 2, 5, and placing one foot in A, with the other describe the pricked Arch *a b*. Likewise take in your Compasses 1, 48 the length of BC, and placing one Foot of your Compasses in C, with the other describe the pricked Arch *c d*, crossing the former Arch *ab* in the

the point B, then drawing 2 Lines from C and A to the point B, you have protracted the Triangle ABC: After the ſame manner muſt you protract the Triangles AFC, FEC, and CED, ſo is your work finiſhed, and the Figure of the Field protracted upon Paper.

CHAP. VI.

Now for to know how to Meaſure this Plot thus protracted, or any of the 2 former, pag. 223, 224, *and by conſequence any other: Take theſe Rules following.*

SECT. I.

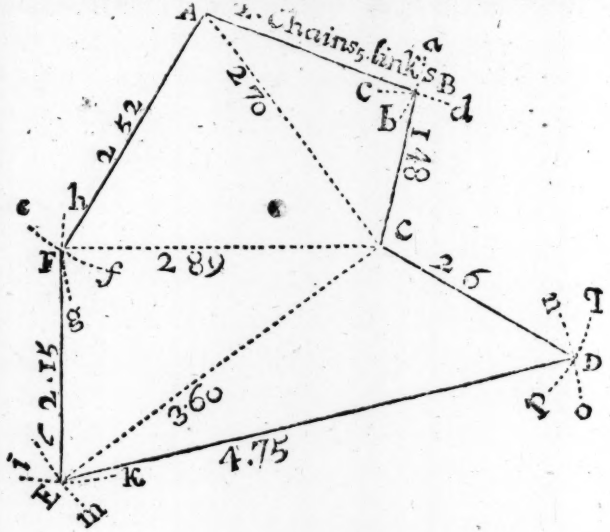
Firſt, You muſt Reduce the ſaid Plot or Figure into one or more of the Three Regular Geometrical Figures before-mentioned, viz. *the* Circle, Square, *or* Triangle.

NOW there are very few, or no pieces of Land which can be reduced to the firſt, *viz.* the *Circle*, but what may be done as to that you have *pag.* 74: But to the ſecond, *viz.* the *Square*, many ſorts of

of Figures may be reduced thereunto, as you may ſee *pag.* 89, 90. But in the third and laſt place, any Irregular Plot whatſoever may be reduced to Triangles, as you may ſee *pag.* 04 and 105, and as you ſee this Plot is done.

SECT. 2.

Now this being done, there is no more to be done, but to meaſure theſe ſeveral Triangles by letting fall perpendiculars, as



is taught *pag.* 55. and multiplying half the Baſe, and the whole Perpendicular, or the whole

whole Baſe and half the Perpendicular together, which will give the ſuperficial Content of each Triangle, and the Contents of the ſeveral Triangles added together, will give the Content of the whole Plot in Acres, Roods, and Perches.

RULE 1.

And for finding the Superficial Content of a Triangle, having all the Three ſides thereof given, it is taught *pag.* 113 which may be done without letting fall any perpendicular: For the doing whereof I refer the Reader to the place before-mentioned, *pag.* 113.

RULE 2.

The Second Rule is, To find how many Acres, Roods and Perches are contained in any Triangle, or any other piece of ground of any other ſhape. And for this purpoſe note, that 3 Barley Corns is an inch, 12 inches a Foot, 3 Foot a Yard, *&c.* And that by the Statute 5½ Yards, or 16½ Foot make a Perch or Pole, 40 ſquare Perches make a Rood, and 4 Rood an Acre, ſo that 160 ſquare Perches are an Acre: Suppoſe a piece of Ground ſhould contain 917 ſquare Perches, Divide 917 by 40, the Quotient will

will be 22 $\frac{37}{160}$, divide this again by 4, the Quotient is 5, 2, 37, which is 5 Acres, 2 Rood, 37 Perches; or divide 917 by 160, the Quotient will be 5 Acres, and 117 Perches, which is 2 Rood, 37 Perches, as before.

RULE 3.

Note again to bring Chains and Links into Acres, Roods, and Perches: That 100 Links (of Mr. *Gunters*) make a Chain, 10 Square chains an Acre: Now ſuppoſe a Piece of Land to contain 9 Chains, 50 Links in length, and 6 Chains 25 Links in breadth, lying in a long Square: Multiply the one by the other, or 950 Links by 625, the Product is 5L93750, from which Product in Square Links cut alwaies off the five laſt Figures, ſo is the reſt to the left hand compleat Acres, which is here 5. Thoſe which are cut off being 93750 multiplyed by 4, (the number of Roods in one Acre) produceth 3L75000, from which Product cutting off again the 5 laſt Figures towards the Right Hand, the Remainder towards the Left Hand ſhews the compleat Roodes, which are here 3: The 5 Figures cut off being 75000, multiplyed by 40 the Number of Perches in a Rood, produceth 30L00000; ſo cutting off the 5 laſt Cyphers towards the

the Right hand, leaves 30 towards the Left, which are 30 Perches: If any odd Numbers had remained in thoſe which were cut off, they had been ſo many Hundred Thouſand parts of a Perch; but no Fraction remaining, the Contents of this Parallelogram or Long Square, is 5 Acres, 3 Roods, and 30 Perches: the like is to be done in any other.

SECT. 3.

But to help you herein, obſerve the Table following as to the former Example 593750. Cutting off the 5 laſt Figures, there remains 5 Acres: Then look for 90000 in the Table under the Title Links, and againſt it under the Title Roods and Perches, you find 3 Roods 24 Perches: Then look for 3750, and againſt it you find 6 Perches, all which added together, gives you the Area or Contents of the whole, as before; 5 Acres, 3 Rood, and 30 Perches. The like is to be obſerved in any other Number of Square Links, to Reduce them to Acres, Roods, and Perches.

	5.	A.	o.	R.	o P.
		3			24
		::			6
Somma	5	3			30

Square

Square Links.	R.	P.
100000	4	00
90000	3	24
80000	3	08
70000	2	32
60000	2	16
50000	2	00
40000	1	24
30000	1	8
20000	0	32
10000	0	16
9375	—	15
8750	—	14
8125	—	13
7500	—	12
6875	—	11
6250	—	10

SquareL.	R.	P.
5625	—	9
5000	—	8
4375	—	7
3750	—	6
3125	—	5
2500	—	4
1875	—	3
1250	—	2
625	—	1

Note that 220 *yds* or 11 Score in Length, and 22 *yds.* in Breadth makes an Acre.

SECT. 4.

Having the Length of any piece of ground in Perches, To find how many Perches in Breadth will make an Acre.

As the Length in Perches (which suppose 50) is to 160 upon the Line of Numbers, the same extent the same way will reach from 1 to the Breadth in Perches in this Example, $3\frac{20}{100}$, or $3\frac{1}{5}$ part of a Perch: Divide

vide 160 by 50, Quotient $3\frac{10}{50}$, which is the ſame. But in Chains and Links: As the length in Chains is to 10, ſo is one Acre to the Breadth in Chains and Links: As the Extent from 12 Chains 50 Links to 10, will reach from 100 Links, or 1 Acre to 80 Links the Breadth required. *Leyburn pag.* 276.

SECT. 5.

To know whether you have taken the Angles of a Field truly.

Add all the Angles together, and if the ſum of them all do agree with the Sum of 180 degrees multiplyed by a Number leſs by two than the Number of Angles in the Field, your work is true, otherwiſe not: This is where all the Angles are inward, but if any of the Angles be outward, then take the Complements of ſuch Angles to 180 Degrees, which is to be added to the reſt of the Angles inſtead of the Angles themſelves, and theſe outward Angles (if never ſo many) muſt not be accounted as Angles in the Multiplication, but wholly rejected.

To Reduce Chains and Links (of Gunters Chain) into Feet and Yards.

Set down the Number of Chains and Links as one whole Number, (as ſuppoſe 5 Chains 32 Links, ſet them down 5.32) multiply this 532 by 66, the Product is 351⌊12, from whence cut the two laſt Figures towards the Right hand, ſo ſhall the Figures to the Left hand remaining be Feet, and the Figures cut off 100 parts of Feet, *viz.* $351\frac{12}{100}$ Feet, Theſe divided by 3, brings them into yards, *viz.* 117 yards. Or multiply 532 by 22, is 117⌊04, and cut off the two laſt Figures, brings it into yards at once, *viz.* 117 as before, and $\frac{4}{100}$ parts of a yard the Fraction. But let the Number of Chains be what they will, if the number of Links be leſs than 10, you muſt place a Cypher before the number of Links, as ſuppoſe 9 Chains 5 Links, place them thus, 9⌊05, then multiply them by 66, as in the other Example, will produce 59730, from which cut off the two laſt Figures, and there will appear to be 597 Feet, $\frac{30}{100}$ parts, and ſo of any other: As for Example.

9.05
66

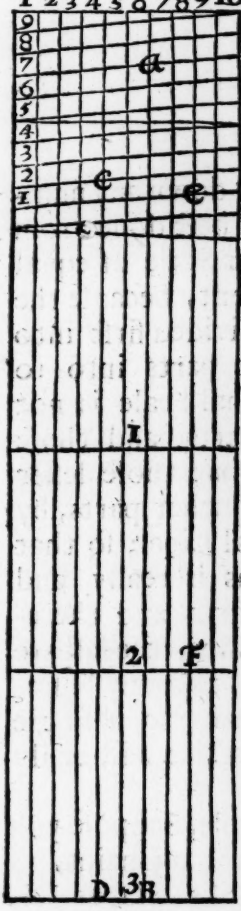
5430
5430

597.30

Note further, that to lay down any number of Chains and Links, is beſt done from a Diagonal Scale, any other Scale of equal parts not being ſo convenient, becauſe the ordinary Scales are only divided firſt into 10 parts, and one of thoſe parts into 10 leſſer parts; but the Diagonal Scale is not only firſt divided into 10 parts, and thoſe again into 10 leſſer parts; but thoſe leſſer parts alſo divided into ſo other parts, by means of the croſs Diagonal Line; ſo that each of the largeſt Diviſions is really and viſibly divided into 100 parts, and therefore taking the firſt for Chains, the laſt are Links, and out of it you may take any Number meaſured by your Chain to a ſingle Link, to which a little practice will eaſily bring you.

As to give you one Example, Suppoſe you were to take from your Scale 3 Chains, 76 Links. Firſt, Take three of the large Diviſions in your Compaſſes for the three Chains:

Chains: Then for the Links obſerve, that the ſmall Diagonal Diviſions are numbred from 1 to 10 upon the Top, and likewiſe from 1 to 10 upon the ſide, now reckon upon the ſide upwards from 1 to 7, and thoſe ſignifie Tens, and run along that Line till you come juſt under 6 on the Top, and thoſe ſignifie Units, and where thoſe two Lines meet, there is the place ſignifying 76. Therefore the Extent of your Compaſſes from *a* to B is exactly 3 Chains, 76 Links, and from D to C is 3 Chains, 24 Links, and from *e* to F is 2 Chains 18 Links; and ſo of any other number that you have occaſion to protract, be the ſame more or leſs: The Rule is the ſame alwaies, counting upon the ſide upwards for Tens, and from the left ſide towards the right you muſt reckon Unites, and obſerving to meaſure Chains and Links alwaies upon the ſame



ſame parallel Line, which is drawn from one end of the Scale to the other.

SECT. 6.

To take the Plot of any Champian Field of 2000, *or* 3000 *Acres by the Plain Table, and never change Paper.*

Place your Inſtrument at every Angle, and ſo get every Angle and his ſides, not regarding the length of the containing ſides as you be wont. Then Meaſure every Hedge, and as you uſe to lay the ſame down by your Scale and Compaſs, here you ſhall but write the length of every Hedge upon the Lines drawn upon your Paper, ſo they need not run off the ſame, becauſe you may draw them as long and as ſhort as you pleaſe, but when you come at home, you muſt protract them according to due proportion.

SECT. 7.

Angles taken by the Interſection of Lines often fall out ſo acute, that it is hard to find the true point of Interſection.

Therefore when you have taken a Field at two Stations by interſection of Lines (as is taught, *pag.* 223.) Then from the firſt Sta-

Station beyond the ſecond, or from the ſecond Station beyond the firſt, obſerve ſome Mark or Tree inclining towards the propoſed Field (which let make rather a Right Angle than obtuſe Angle with your Stationary Line) and ſo get the Angles it makes with both your Stations, making your Stationary Line one ſide of the ſaid Angles: Then go to the mark eſpied for a third Station, and there obſerve ſuch Corners that you thought would fall out Acute by Sections of Lines iſſuing from the firſt ſtations, and thereby get the Angles they make with your firſt and ſecond ſtations; for by help thereof, you ſhall correct the acute Section of the former Lines.

And note, that where a Field lies ſo that you cannot from 2 Stations ſee all the Corners thereof, what you cannot do at the firſt and ſecond ſtation, you may do with as many more ſtations as need requires, as a third, fourth, or fifth Station, or more, as you ſee cauſe.

SECT. 8.

To take the Plot of a Wood, or Wateriſh Ground, when by Reaſon of ſome obſtruction or other, you cannot come within it.

Your way is to go round about the ſame, and

and make your obſervations at every Angle, the Diſtance between Angle and Angle meaſured by your Chain will be the ſides of the Plot, and protracting both the Angles and Sides upon Paper, will give you the true Figure of the whole, which you may meaſure by the Rules aforegoing.

SECT. 9.

Take notice, that the moſt, or however, very many ſmall incloſures are Trapezia, that is, having 4 unequal Sides, and 4 unequal Angles: Now any of theſe may be eaſily reduced into two Triangles, and ſo meaſured as I have formerly ſhewed you: But for the more ready caſting up the Contents of theſe, note, that the Diagonal Line drawn from Corner to Corner, whereby the Trapezia is divided into 2 Triangles, is by that means the common Baſe to them both. Therefore add the two perpendiculars together, the half of which multiplyed by the whole Baſe, the Product will ſhew the Contents of the whole Trapezia; or if you add both the perpendiculars together, and by that multiply the whole Baſe; half of the Product will be the Content of the Trapezia.

SECT. 10.

Another way to meaſure a Field of 4

unequal ſides is this: Firſt, Meaſure the top and the bottom, and add thoſe two together, and take the half, then meaſure the two oppoſite ſides, adding them alſo together, and take the half, Theſe two halfs multiplyed the one by the other, the Product will give you the true Contents: As ſuppoſe one ſide be 16 Perches, its oppoſite ſide 6, theſe added, makes 22, the half whereof is 11: Then ſuppoſe one of the other ſides 28, its oppoſite ſide 18, theſe added is 46, the half whereof is 23, which multiplyed by 11, ſhews 253 to be the whole Content. Note, that to protract a Trapezia, you muſt by the Theodolite, or ſome ſuch like Inſtrument obſerve one of the Angles, which being known and protracted, prick off the 2 containing ſides, and from the end or the Extremities of thoſe 2 ſides, you may draw the other two ſides by 2 Arches, croſſing one another at their Reſpective Diſtances, as in a Square or Triangle, as is elſewhere taught.

A Table

A TABLE of *FRACTIONS.*

O R,

Parts of Integer Pearches, that is, in any Number of Pearches, so far as the Table Extends. What a Quarter of a Pearch awanting, or over, will amount unto in Dayworks, Pearches, Yards, Foot, and Inches Square.

A Quarter of a Pearch in Pearches.		D.	P.	Y.	F.	I.	A Quarter of a Pearch in Pearches.	D.	P.	Y.	F.	I.
P. 1		0	0	1	1	$1\frac{1}{4}$	P. 21	1	1	1	1	$1\frac{1}{2}$
2		0	0	2	2	3	22	1	1	2	2	3
3		0	0	4	0	$4\frac{1}{2}$	23	1	1	4	0	$4\frac{1}{2}$
4		0	1	0	0	0	24	1	2	0	0	0
5		0	1	1	1	$1\frac{1}{4}$	25	1	2	1	1	1
6		0	1	2	2	3	26	1	2	2	2	3
7		0	1	4	0	$4\frac{1}{2}$	27	1	2	4	0	$4\frac{1}{2}$
8		0	2	0	0	0	28	1	3	0	0	0
9		0	2	1	1	$1\frac{1}{4}$	29	1	3	1	1	$1\frac{1}{2}$
10	is	0	2	2	2	3	30	1	3	2	2	3
11		0	2	4	0	$4\frac{1}{2}$	31	1	3	4	0	$4\frac{1}{2}$
12		0	3	0	0	0	32	2	0	0	0	0
13		0	3	1	1	$1\frac{1}{2}$	33	2	0	1	1	$1\frac{1}{2}$
14		0	3	2	2	3	34	2	0	2	2	3
15		0	3	4	0	$4\frac{1}{2}$	35	2	0	4	0	$4\frac{1}{2}$
16		1	0	0	0	0	36	2	1	0	0	0
17		1	0	1	1	$1\frac{1}{4}$	37	2	1	1	1	$1\frac{1}{2}$
18		1	0	2	2	3	38	2	1	2	2	3
19		1	0	4	0	$4\frac{1}{2}$	39	2	1	4	0	$4\frac{1}{2}$
20		1	1	0	0	0	40	2	2	0	0	0

A Quarter of a Pearch, in Pearches.

P.		D.	P.	Y.	F.	I.
41		2	2	1	1	1 1/3
42		2	2	2	2	3
43		2	2	4	0	4 1/2
44		2	3	0	0	0
45		2	3	1	1	1 1/2
46		2	3	2	2	3
47		2	3	4	0	4 1/2
48		3	0	0	0	0
49		3	0	1	1	1 1/2
50	is	3	0	2	2	3
51		3	0	4	0	4 1/2
52		3	1	0	0	0
53		3	1	1	1	1 1/2
54		3	1	2	2	3
55		3	1	4	0	4 1/2
56		3	2	0	0	0
57		3	2	1	1	1 1/2
58		3	2	2	2	3
59		3	2	4	0	4 1/2
60		3	3	0	0	0

A Quarter of a Pearch in Pearches.

P.	D.	P.	Y.	F.	I.
61	3	3	1	1	1 1/2
62	3	3	2	2	3
63	3	3	4	0	4 1/2
64	4	0	0	0	0
65	4	0	1	1	1 1/2
66	4	0	2	2	3
67	4	0	4	0	4 1/2
68	4	1	0	0	0
69	4	1	1	1	1 1/2
70	4	1	2	2	3
71	4	1	4	0	4 1/2
72	4	2	0	0	0
73	4	2	1	1	1 1/2
74	4	2	2	2	3
75	4	2	4	0	4 1/2
76	4	3	0	0	0
77	4	3	1	1	1 1/2
78	4	3	2	2	3
79	4	3	4	0	4 1/2
80	5	0	0	0	0

It you have occaſion for half Pearches, take theſe Sums twice; if for three quarters take the ſame Sums thrice, and the total will be your déſire.

Note that 3 Barly Corns is an Inch, 12 inches a Foot, and 3 Foot a yard, 5 1/2 Yards a Pearch, 4 Pearches a Day-work, 10 Day-work a Rood, 4 Rood or 40 Pearches an Acre: This

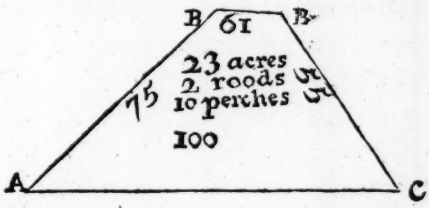
This Table may be very uſeful for thoſe that have their Chains and Meaſuring Lines divided only into Pearches, half Pearches, and Quarters: But the moſt exact way is, to caſt up by Chains and Links for ſuch as have Mr. *Gunters* Chain, which way you have taught, *pag.* 231, 232. But I leave every one to uſe which way pleaſes him beſt, and ſo conclude *Surveying*.

As an Appendix I ſhall add theſe 3 Propoſitions following.

PROP. I.

How to take the Superficial Contents of Hills and Valleys according to Mr. Diggs.

Suppoſe A B C were a Mountain, firſt, meaſure the Circuit of the Baſe A C 100 Pearches: Then meaſure the Compaſs of the Summity or Top B B 16 Pearches,



which added together, makes 116 pearches: Then meaſure the aſcent of the Hill AB 75

pearches, and its oppoſite Aſcent C B 55 pearches, adding theſe two Aſcents together 130, half of which is 65, which multiplyed by Half of the Circuit 58, the Content will be 23 Acres, 2 Roods, and 10 pearches the true content of the Hill.

For meaſuring a Valley inſtead of the Circuit at the Baſe in a Mountain, you muſt meaſure the Circuit of the top of the Valley, and inſtead of the Circuit of the top of the Mountain, you are to meaſure the Circuit of the depth of the Valley, and inſtead of the Aſcenſes of the Hill, meaſure the Deſcenſes of the Valley, which makes it an hollow hill turn'd upſide down, and to be meaſured by the Rule aforegoing.

PROP. 2.

To find the Horizontal Line of a Hill or Mountain.

This is no more but reſolving a Hill into 2 right Angled Triangles, and then to find the Baſe of both thoſe Triangles which is the Horizontal Line, or the Line on which the Hill ſtandeth. Now to do this, place your Quadrant at the foot of the Hill on one Side, and through the Sights eſpying the top of the Hill (or a mark ſet as high above it, as your Inſtrument ſtands above the ground) Obſerve what degrees are cut by the Thread upon the Limb of

the

the Quadrant (ſuppoſe 47 *d.*) then the Angle at the top muſt be the Complement thereof to 90, *viz.* 43 *d.* then meaſure the diſtance betwixt the top and the bottom, which ſuppoſe 71 feet, being the Hypothenuſal of the Triangle oppoſite to the right Angle 90, ſo having theſe given, you may eaſily find the Baſe and the perpendicular alſo, if required: For as 90 Degrees is to 71 Feet, ſo is 47 *d.* to the perpendicular, or 43 to the Baſe required. Now having got this part of the Horizontal Line, you muſt get the other part by proceeding after the ſame manner from the other ſide of the Hill, taking the Angles, meaſuring the Aſcent, and finding the Baſe as above, and theſe two Baſes added together, makes up the true Horizontal Line of the Hill or Mountain.

PROP. 3.

To find the Level betwixt any two places, and whether Water may be conveyed from a Spring-head, to any appointed place.

There is an Inſtrument called a Water-Level whereby this is beſt effected, though it may be alſo done by a Quadrant: For the Water-Level it is only a ſmall Pipe of Glaſs made cloſe at both ends with Wax,

and filled with Water, excepting a very ſmall vacuity, which will ſhew it ſelf in the middle, when it lies Level; but upon the leaſt leaning either way, you will find it move accordingly. Now this laid in a piece of Wood, having ſights at either end, is a fit Inſtrument for this purpoſe: This Water-Level placed at any convenient diſtance from the Spring-head in a right Line towards the place to which the Water is to be conveyed as 50 or 100 yards, and having two long Staves, cauſe one of the Staves to be ſet up at the Spring head, the other let be erected as many yards beyond your Inſtrument towards the place to which the Water is to be conveyed: Theſe Station Staves erected perpendicular, and your Water-Level preciſely horizontal, look through the Sights to the firſt ſtaff, cauſing a Leaf of Paper to be run up and down the Staff till it be eſpied through the Sights, there obſerve how many Feet, Inches, or Parts the Paper reſteth upon, which note in your Book: Your Water-Level remaining immoveable, go to the other end of it, and cauſing a paper to be moved up and down the other Staff as before, note how many Feet, Inches, and Parts the paper reſteth upon: Then if the ſecond ſtaff be more than the firſt, deduct the leſs from the greater, and the Remainder will ſhew how

how much the ſecond ſtaff is lower than the firſt, and ſo by making ſeveral Stations you may find your deſire at what diſtance you pleaſe.

PROP. 4.

Another way by the Quadrant is this, Firſt, Go to the place whither you would convey your Water, and let any perſon ſtand toward the Spring-head any Number of Yards, and obſerve the top of a ſtaff in his hand equal to your eye by the Quadrant: Note the Degrees, and ſo take the Height as before at ſeveral other Diſtances, till you come to the Spring-head it ſelf, and protracting the ſeveral Angles of Altitude as you took your Stations and your diſtance of yards upon the Ground betwixt Station and Station: And in concluſion, you ſhall have the difference of Height betwixt the Spring-head, and the place whereunto you deſire to have your Water conveyed. This is of excellent uſe to find whether Coals may be ſaugh'd or drain'd of Water. Firſt, Boaring, or otherwiſe finding the Depth of the Coal from the ſurface of the Ground, and ſubſtracting that Height from the height found betwixt the place above ground where the Coals lye, and the place whither you deſire to bring your Water: All Coal be-

ing obſerved to lay true Level, let the ſurface of the Ground riſe or fall as it will, excepting where it is extraordinarily thrown up or down by that which they call an Horſe. This therefore may be done either of theſe waies aforementioned, that is, either by the Water-Level, or Quadrant, only obſerve, that at every Miles end there ought to be allowed $4\frac{1}{2}$ Inches more than the ſtrait Level for the Currant of the Water, which in a ſhort diſtance is not conſiderable.

OF

OF NAVIGATION.

CHAP. VII.

HAving now finiſhed what I intend to ſay concerning *Surveying*, and taking of Heights and Diſtances at Land. I come now to treat of that other part of *Geometry*, which is, the taking of Heights and Diſtances at Sea, to wit, *Navigation*: And I hope this Diviſion of Geometry into *Surveying* and *Navigation*, and the Definition of them to be taking of Heights and Diſtances, the one at Land, and the other at Sea; may not by the Candid Reader be thought incongruous or improper, but proper and Genuine to the preſent purpoſe for making them plain and obvious to the Capacity of the meaneſt; which is the principal aim and deſign of this ſmall Treatiſe. But to proceed, the whole Science of Aſtro-

nomy

nomy is a great Ornament and Accompliſhment to him that profeſſes *Navigation*; but of abſolute Neceſſity *Navigation* muſt borrow of *Aſtronomy* the taking of Heights, *viz.* in theſe two particulars, (that is) Firſt, The taking of the Height, or the Meridian Altitude of the Sun: And thereby in the ſecond place, to find the Height or Elevation of the Pole: Yet this latter, in the ſame ſenſe that it is called Latitude, may be the finding or taking of a Diſtance in *Navigation*, being indeed no more but the Diſtance *North* or *South* from the Equinoctial, or any other ſuppoſed parallel; as Longitude is the Diſtance *Eaſt* and *Weſt* from the firſt Meridian, or any other ſuppoſed Meridian: And as the Rumb leading from one place to another may be called the Diſtance run upon ſuch a point of the Compaſs. Now the taking theſe Heights and Diſtances is the Sum and Subſtance of the Art of *Navigation*. And to Treat of theſe particulars in order, ſhall be the method of my intended Diſcourſe upon this Subject.

Now theſe may be more ſuccinctly comprized in theſe four Terms, being the ſame which are uſed by *Norwood*, *Hues*, and others. To wit,

1. The

1. The Difference of Latitude.
2. The difference of Longitude.
3. The Rumbs.
4. The Diſtance run upon the ſaid Rumb.

Any two of theſe being known or given, the other two may alſo be found; whence obſerve, That there is a two-fold finding of each of theſe. Firſt, As they are *Data*, to be given or known, and ſo they are found by obſervation. Secondly, As they are *Quæſita*, ſought or required, and ſo they are found by *Trigonometry*, or *Arithmetical Calculation* in the Reſolution of a Plain Right Angled Triangle; ſo that in concluſion you will find that *Navigation* as well as *Surveying* is ſtill no more but the Reſolution of a Triangle: But to proceed Methodically: Seeing there muſt be alwaies two of theſe given or known, to find the reſt: It will be very neceſſary to ſhew you how in the firſt place they may be found as *Data* by Obſervation.

SECT.

SECT. 1.

And first for finding the Latitude or the Elevation of the Pole.

PROBL. 1.

This is done by observing the Meridian Altitude of the Sun or Stars either by an Astrolabe, Cross-staff, or Quadrant.

How to take the Meridian (or any other) Altitude of the Sun or Stars by the Cross-staff by forward Observation, is the same as in the General use of that Instrument in taking of Altitudes, as is taught *pag.* 212,213: And how to take the same Altitude by Thread and Plummet, and backward observation, is also taught particularly *pag.* 212. §. 4. But the easiest and readiest way to take the Meridian, (or other) Altitude of the Sun is, to do it by the Astrolabe or Quadrant thus, by that which is called backward Observation, and without looking through the sights: Let the Sun shine through that sight which is next the Center, and bring that Beam to fall right upon the hole of the other sight, and the Thread will fall upon the true Altitude in the Quadrant, or in the same manner the

Index

Index in the Aſtrolabe will cut the Degrees of Altitude.

Now to find the Meridian Altitude of the Sun, (though the Sun do not ſhine) may be done by knowing the Declination and Latitude.

For if the Declination be North, you muſt add it to the Complement of the Latitude, (which is alwaies the ſame with the Height of the Equinoctial;) and if the Declination be South, you muſt ſubſtract it from the Complement of the Latitude, which will give you the Meridian Altitude at any time. As ſuppoſe at *London* the Elevation of the Pole (or the Latitude) is 52*d.* the Complement whereof to 90 *d.* is 38 *d.* (which is alſo the Height of the Equinoctial.) Now the ſecond of *May* the Sun being in 20 *d.* 42 *m.* of *Taurus*, his declination Northwards is 117 *d.* 56′. 21″. which added to 38, makes the Meridian Altitude of the Sun to be 55 Degrees, 56 Minutes, and 21 Seconds.

If it be at the time of the Equinoctial, (*viz.* 11*th* of *March*, or the 13*th* of *September*) the Height of the Sun or Star (when they are upon the Meridian) ſubſtracted from 90 Degrees, will ſhew the true Latitude; but at any other time you muſt find out the Declination of the Sun or Stars, and

and if the Declination be Northerly, then ſubſtract it from the Altitude: If Southerly, you muſt add it to the Altitude; by which Addition or Subſtraction you ſhall have the Height of the Equinoctial above the Horizon, and that being ſubſtracted from 90 Degrees, will ſhew the true Latitude of the place where you then are.

PROBL. 2.

To find the Elevation of the Pole by the Globe.

Firſt, Taking the Meridian Altitude of the Sun, and bring the place of the Sun in the *Ecliptick*, or the Star to the *Brazen Meridian*, and ſo move the *Brazen Meridian* together with the Globe through the Notches it ſtands in, till the place of the Sun, or the Star be elevated ſo many Degrees above the Horizon, as the Suns or Stars Meridian Altitude is: The Globe ſtanding in this poſition, the Pole will be elevated according to the true Latitude of the place. *Example,*

The 12*th* of *June* I find the Suns place in the beginning of Cancer, and the Meridian Altitude of the Sun at *London* 62 degrees; bring the firſt degree of Cancer to the Meridian, and elevate the ſame 62 degrees above the Horizon, ſo you will find the Pole elevated at *London* 51 Degrees, 30 Minutes:

Minutes: Or by the former Rule, having the Suns Meridian Altitude 62, and finding by the Table of the Suns Declination or otherwiſe, that the 12*th* of *June* the Sun hath his greateſt Declination, *viz.* 23 *d.* 30 *m.* ſubſtract this from 62 *d.* reſts 38 *d.* 30 *m.* for the Height of the Equinoctial; and that again ſubſtracted from 90 *d.* leaves 51. 30 for the Latitude or Elevation of the Pole.

And ſeeing the finding of the Latitude by the *Sun* depends upon knowing his place in the Zodiack, or his Declination; and if you would find the Latitude by the ſtars, you muſt alſo know their Declinations. Therefore I ſhall give you two ſhort Rules for the two former, and a ſhort Table for the latter in this place, but muſt refer the Reader to larger Rules and Tables when I come to ſpeak of them in Aſtronomy.

PROBL. 3.

To find the Suns place by theſe two Verſes.

♈ ♉ ♊ ♋ ♌ ♍
Evil, attends, its, object, unvail'd, vice,
♎ ♏ ♐ ♑ ♒ ♓
Vain, Villains, jeſt, into, a, Paradiſe:

Theſe 12 words ſignifie the 12 months beginning

ginning at *March*, and the 12 ſigns in order the Sun enters into every Moneth, and to know what day of the Moneth he enters: If the firſt Letter of the word proper to the Month be a Conſonant as Paradiſe belonging to *February*, in that Moneth he enters the 8*th* day, but if it be a Vowel, (as all the reſt are) add ſo many days to 8, as the Vowel denotes in order of the 5 Vowels a e i o u } which will ſhew what day of the 1 2 3 4 5 } Moneth the Sun enters any of the 12 ſigns, and reckoning that the ſun goes thorough one Degree every day, you may find the Degrée the ſun is in, and (by the hours of the day from Noon) the Minutes of that degree.

PROBL. 4.

To find the Suns Declination by Logarithmes, or the Lines of Artificial Sines.

As Radius or 90 Degrees is to the *Suns* greateſt Declination (which is alwaies 23 Degrees 30 Minutes) *So* is (the Number of Degrees or Minutes from the next Equinoctial point to the ſuns place) to the ſuns preſent Declination: *Suppoſe* the ſuns place be 0 degrees, or the beginning of *Taurus*, that is, 30 degrees from the next Equinoctial

ctial point of Aries, his Declination will be found to be 11 *d.* 30 *m.* for as 90 to 23, 30, ſo 30 to 11, 30, the like Proportion will hold in any other.

PROBL. 5.

To find the Declination of ſome notable Stars, obſerve this Table.

	North			South	
The firſt Horſe in the wain.	57	27	Spica Virginis.	8	55
The third in the wain.	51	5			
Arcturus.	21	53	Heart of Scorpion.	25	30
The Eagle.	7	57	Orions right ſhoulder.	6	16
The Bulls Eye.	15	54			
Caſtor.	32	30	Orions left foot.	9	10
Pollux.	28	30	The great Dog.	15	56
The Lyons Heart.	13	40	The little Dog.	5	53
Theſe ſtars have North Declination.			Theſe have South Declinations.		

Thus much for Latitude.

SECT. 2.

In the next place, for finding the Longitude.

If the Longitude could be as certainly found as the Latitude, it would render *Navigation* perfect; but notwithſtanding that many have pretended thereunto, yet none

none have attained to it in reality as far as I can yet find: *Theaker* in his Planiſphere, which he calls a light to the Longitude, tells you he can by that Inſtrument as certainly do it, as to find the Riſing, Setting, and Southing of the Stars, but if he can, he does very ill expreſs it, and I muſt conclude, that, *aliquid latet quod non patet*, and perhaps he might keep a reſerve to himſelf on purpoſe.

The moſt Rational waies of finding the Longitude which I meet with, are theſe 4, Collected out of *Varenius*, and *Carpenters* Geography.

PROBL. 1.

Firſt, By an Eclipſe of the Moon.

By obſerving how much ſooner the eclipſe beginneth at a place of known Longitude, (which you may find by an Ephemerides) than at the place where you are; where you may find the time of the Eclipſe by obſervation with the Teleſcope, and knowing the Latitude at the ſame time by the Stars, you may find the true hour of the Night. This done, the difference of this time of the Moons entrance into the Eclipſe, or the middle or ending of the ſame at the place where your obſervation was made, being

being converted into Degrees and Minutes, and either added or ſubſtracted from the hour of the beginning, middle, or end of the Eclipſe at the other place of known Longitude found by the Ephemerides: I ſay the Degrees and Minutes of the difference betwixt the hour at one place, and the hour at the other place added or ſubſtracted from the Degrees and Minutes of the known Longitude, gives you the Longitude of the place required.

PROBL. 2.

By a Clock, Watch, or Hour-glaſs.

If you have a very true Watch, and be to ſail from a place of known Longitude, when you depart from that place, by an Aſtrolabe, or otherwiſe by the Sun or Stars obſerve the hour of the day or night, and ſet your Watch exactly to the ſame hour, when you have a mind to know what Longitude you are in, obſerve again by the Aſtrolabe, or otherwiſe the exact hour; and if the Watch agree with the obſervation, there is no difference of Longitude; or what difference betwixt theſe two you find added or ſubſtracted to or from the Longitude of the place you came from, ſhews the Longitude of the place you are

now

now at: This way you ſee depends upon the exact taking the hour at both places, and upon the exactneſs of the Watch; therefore in this latter reſpect an Hour-glaſs is accounted fitter for Sea.

PROBL. 3.

Thirdly, By obſervation of the Difference in the Sun and Moons motion.

Firſt, You muſt take for granted, that the Motion of the Moon is 48 minutes of an hour ſlower in 24 Hours, or 360 Degrees than that of the Sun. Secondly, You muſt ſuppoſe that by Mathematical helps a man may know in any place the Meridian, (or South point) and alſo the hour of the day, and at leaſt by an Ephemerides the time of the Moons coming to the South, or to the Meridian. Now ſuppoſe at *London* you find (by an Ephemerides calculated for that place) that the Moon comes to the Meridian (on ſome ſet day) at four a Clock after Noon: And you being in the *Weſt-Indies* the ſame day you obſerve the Moon come to the Meridian at 10 Minutes after four: Then ſay by the Rule of Proportion, If the difference of the Sun and Moons motion be 48 Minutes of an hour in 360 Degrees, what will it be in 10 Minutes: Or if 48 give 360,

360, what gives 10? The fourth proportional Number will be 75, which is the diſtance of that place from *London*, from which number the Longitude of *London* being ſubſtracted (20 degrees) and the Remainder (55) again ſubſtracted from 360, the Reſidue will ſhew the Longitude (305).

PROBL. 4.

By the Diſtance betwixt the Moon, and ſome known Star ſcituate near the Ecliptick.

But this, and ſeveral other waies mentioned in *Varenius* as by the Moons place, her entrance into the Ecliptick, *& per Planetas Joviales:* I ſhall not trouble the Reader with, but refer him to the Author, 418, 420, 421, and paſs on. Now the Difference of Latitude, or the difference of Longitude, is ſo many Degrees, Minutes, or Miles as the Latitude or Longitude of one place differs from the Latitude or Longitude of another place, the Latitude or Longitude being obſerved or known at both places; as knowing the Latitude of the place from whence you ſet Sail, and having ſailed a certain number of Leagues, there you take an obſervation of the Latitude as is beforementioned: Now if both the places be on the *North-ſide* the Equinoctial, or

on

on the ſouthſide the ſame ſubſtract the leſſer latitude from the greater, the remainder is the difference of Latitude: If one of the places be under the Equinoctial, and the other on the North or South ſide thereof, the difference of Latitude is the Latitude it ſelf. If one of the places be on this ſide the Equinoctial, and the other on the other ſide, the Sum of both the Latitudes added together is the difference of Latitude, or the Diſtance North and South betwixt the two places; and if the Longitude could be as exactly found, it would ſhew their Diſtance Eaſt and Weſt; However the difference of Longitude is the Diſtance of any place upon the Line of Eaſt and Weſt from the Meridian of any other place propoſed, but becauſe this cannot certainly be found, or however not without much difficulty by obſervation it is ſeldom given, but required in *Trigonometry*, and found by the Reſolution of a Triangle, as ſhall be ſhewed hereafter in its due place, and ſo much for the two former particulars, the Difference of Latitude and Longitude.

SECT. 3:

In the third place we come to ſpeak of the third conſiderable in Navigation, *viz. the* Rumb.

Which in plain *Engliſh* is the point of the

the Compaſs upon which the ſhip Sails, and this is uſually one of the *Data's*, or things given, becauſe the Sea-mans Compaſſe by which it is directed or ſteered alwaies ſhews this. The finding out of that moſt admirable vertue of the Magnet, being the greateſt advantage that ever hapned to *Navigation*: Indeed the difficulty lies in this, that the Needle touched by the Loadſtone in ſeveral places admits of ſeveral Variations, which muſt be carefully obſerved and taken notice of.

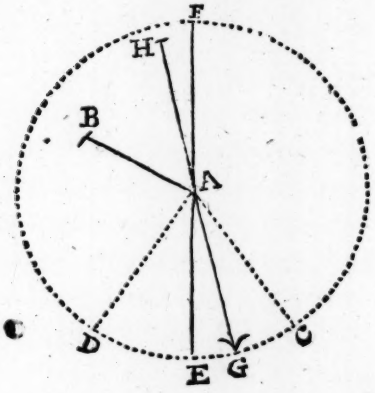
SECT. 4.

To find this Variation of the Needle or Compaſs.

There are ſeveral waies, As by the *Azimuth Compaſs*, and by the *Sea-Rings* deſcribed by Mr. *Wright*; but theſe are chargeable. I ſhall in this place only ſhew you how to do it theſe three waies, the one at Land, the other will ſerve both at Sea and Land.

I. *To find the Variation of the Compaſs, and withal, a true Meridian Line.*

Firſt, Deſcribe ſeveral Circles upon a Plain, and in the Centre place a *Gnomon* or Wire perpendicular as AB. In the Fore-



noon obſerve when the Extremity of the ſhadow of AB juſt touches one of the ſaid Circles, and make there a mark at C. In the Afternoon obſerve again, when the extremity of the ſhadow of AB toucheth the ſame Circle, making there another mark at D. Divide the Diſtance CD into 2 equal parts, which ſuppoſe at E draw the Line EAF, which is the Meridian Line: Then place a Needle GH upon the Center, and mark how many degrees the point of the Needle G is from E, ſo much doth the Needle vary from the North in that place.

II. *To find the Variation of the Needle by the Globe either at Sea or Land.*

Firſt, Rectifie the Globe, and bring the Suns

Suns place to the Eaſt ſide of the Horizon, and look in the Circle of Winds, and againſt the place of the Sun you have the point of the Compaſs upon which the Sun riſeth: Then obſerve by the Compaſs what point of the Compaſs the Sun riſeth or ſets on Morning or Evening, and ſee what difference there is by the Globe and by the Compaſs, and that is the Variation (if any be.) Allowing for this variation, the Needle will alwaies ſhew the Rumb, or the true point of the Compaſs upon which the Ship is ſteer'd.

III. *To find the Suns Amplitude, and thereby the Variation of the Compaſs.*

As the Co-ſine of the Latitude is to Radius, ſo is the ſine of the Declination to the ſine of the Amplitude. Suppoſe the Latitude 51 *d.* 32 *m.* its Co-ſine or Complement is 38 *d.* 28 *m.* the Suns Declination 15 *d.* 10 *m.* the Amplitude will be found to be 24 *d.* 52 *m.* North, becauſe the Declination is North. Now the Circumference of the Compaſs divided into 360 *d.* obſerve at the Suns riſing and ſetting how many degrees the Sun is from the direct point of the Amplitude, and ſo much doth the Needle varie in that place. This Obſervation is beſt to be made, when the lower edge

of the Sun ſeems juſt to touch the Horizon.

SECT. 5.

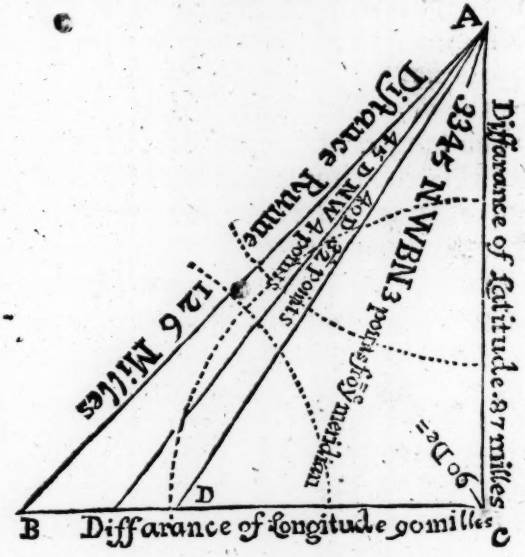
In the fifth and laſt place, To find the Diſtance Run.

This is to know what way the Ship mskes in ſuch a ſpace of time, or how many Leagues or Miles the Ship runs in an hour, *&c.* And this is uſually obſerved by the Log-Line and Minute-Glaſs; ſo many knots as the Ship runs in half a Minute, ſo many Miles ſhe ſaileth in an hour; the ſpace between every knot being 41$\frac{1}{2}$ Feet, reckoning 12 Inches to a Foot, 5 Foot to a Pace, and 1000 Paces to an *Engliſh* Mile: But enlarging the Mile according to Mr. *Norwoods* Obſervation, the diſtance between every knot muſt be 50 Foot, and then as many of thoſe as run out in half a Minute, ſo many Miles or Minutes the Ship ſails in an hour, and for every Foot more the tenth part of a Mile: And if you pleaſe, you may bring theſe Miles into Leagues according to the uſual Cuſtom of Sea-men; and every time the Log is caſt, it ought to be noted upon the Log-board what way the Ship makes. Now beſides the Log-line, there hath been ſeveral other Engines invented, ſome with Wheels, and ſome other

other waies for this purpoſe; As I have ſeen a wheel to Meaſure Land withal, and may be applyed to know how many Miles a Coach goes in an hour, or the like. And for to meaſure any ſmall quantity of time, I find no better an Invention than that of a Swing (being proved by Experience) that a Bullet &c. hung in a ſtring, and being moved from the perpendicular to any Angle (keeping the ſtring ſtrait) and ſo let go to have its own Swing, will keep ſuch exact time in its ſwinging, that though it fetch a larger Compaſs at firſt than afterwards, yet it keeps the very ſame time in every Swing, only the length of the ſtring is the thing that alters the time; and therefore if you would have it go faſter or ſlower, you muſt make your ſtring ſhorter or longer: As a ſtring of 38½ Inches long, (meaſuring from the Center of the Plummet) will exactly make 60 ſwings in a Minute of an hour, and ſo proportionably if you would have fewer ſwings in the time, you muſt lengthen the ſtring, or if more, you muſt ſhorten it. This is meant when the ſtring is faſtned to ſomething in the top of a Room, or ſo: But if you hold it in your hand, it is obſerved that 31½ Inches in length will do the ſame as 38½ faſtned to any ſteddy thing. This may very properly be uſed inſtead of the half Minute glaſs aboard a Ship.

SECT. 6.

Thus you ſee how you may find by obſervation the Latitude and Longitude, the Rumb, and the Diſtance run: Any two of which being known (by reaſon of the Right Angle, which is alſo alwaies known:) The other two may be eaſily found by *Trigonometry*, or the Reſolution of a Plain Right



Angled Triangle, as you may ſee in the Triangle ABC: The Hypothenuſe alwaies repreſenting the Diſtance run AB, the Angle at the North point A repreſents the Rumb,

Rumb which (in the Triangle DAC is North-Weſt and by North:) in the Triangle BAC is North-Weſt: The Baſe BC is the difference of Longitude or Departure from the Meridian; the perpendicular AC is the Difference of Latitude, and the Angle at C you ſee is a Right Angle; and the Angle at B ſhould be the complement of the Rumb.

Now the Tranſmutation of the things to be granted or known, and to be enquired after in theſe four Terms, (*viz.* the Difference of Latitude, the difference of Longitude, the Rumb, and the Diſtance run) may be propoſed ſix manner of waies, and every Propoſition reſolved by the Reſolution of this plain Right-Angled Triangle, as follows in Sailing by the *Plain Cart.*

PROP. 1.

The Difference of Latitude AC and Difference of Longitude BC being known, The Rumb and Diſtance *i. e.* the Angle A, and the Hypothenuſe AB may be found.

This is the ſame as having two ſides and a Right Angle betwixt them, to find the other two Angles, and the third ſide, as is taught *pag.* 137. Or by Protraction, thus:

Firſt, lay down the Baſe 90 Miles, being the difference of Longitude BC, and upon the point C protract an Angle of 90 *d.* laying down the perpendicular 87 A C, being the difference of Latitude, and to touch the ends or extremities of both theſe draw the Hypothenuſe or Diſtance run 126 A B, ſo ſhall the Angle at A be the Rumb 45 *d.* *vide pag.* 132.

PROP. 2.

The Difference of Longitude BC, and the Rumb A being known; The difference of Latitude, ſide AC, and the Diſtance run, *s.* AB. may be found.

This is the ſame as to have 2 Angles and a ſide given, to find the other two ſides: which is done by the general Rule, *pag.* 141. For as the Angle at A, which repreſents the Rumb 45 is to the Baſe BC 90 Miles, ſo is the Right Angle at C 90 *d.* to the Diſtance *s.* AB 126 Miles, and ſo is the Complement of the Rumb, *viz.* 45 at the Angle B to the perpendicular or difference of Latitude 87 Miles AC, either to be wrought by Logarithmes, *pag.* 31. or by the Lines of Numbers, Sines, and Tangents,

gents, *pag.* 41. or by Protraction, *pag.* 132. to which places I refer the Reader.

PROP. 3.

The difference of Longitude BC, and the Diſtance run *ſ.* AB. being known.

The Difference of Latitude *ſ.* AC, and the Rumb Angle A, may be found.

Suppoſing the Right Angle at C 90 *d.* to be known with the other 2 ſides BC and AB. This is the ſame, as having two ſides and an Angle oppoſite to one of them, to find the other Angle oppoſite to the other ſide given, and the third ſide. Therefore by the Rule of 3 ſay, as 90, or Radius is to *ſ.* AB. 126, ſo is the *ſ.* BC. 90 Miles to the Angle A 45 *d.*

PROP. 4.

The Difference of the Latitude AC, and the Rumb, the Angle at A being known,

The difference of Longitude BC, and the Diſtance run *ſ.* AB. may be found.

In this Propoſition (ſuppoſing the Right Angle alſo given) you have all the three Angles given, the third Angle being the

Complement of the Rumb to 90, *viz.* 54 *d.* Therefore ſay by the Rule of Proportion, As the Complement of the Angle at A, *viz.* the Angle B 45 *d.* is to the *s.* AC, ſo is the Rumb or Angle at A 45 *d.* to the *s.* BC, and ſo is Radius or 90 *d.* to the *s.* AB.

PROP. 5.

The Difference of the Latitude AC, and the Diſtance run AB, with the Right Angle C being known,

The Rumb A, and Difference of Longitude B C, may be found.

This is to have two ſides and an Angle oppoſite to one of them given, to find the Reſt, as in Prop. 3. As the Angle C is to *s.* A B, ſo is *s.* AC to the Angle B, and ſo is its Complement A to *s.* BC, which Complement is the Rumb, and the ſide B C is the difference of Longitude required.

PROP. 6.

The Rumb, Angle at A, and the Diſtance run AB, with the Right Angle being known,

The Difference of Longitude BC, and Difference of Latitude AC. may be found.

This

This Propoſition is of all other the moſt uſeful, theſe being the moſt uſual *Data*'s to find the other; moſt Sea-men keeping an Account only of the Rumb or Point of the Compaſs ſail'd upon; and the Diſtance run by often caſting the Log-line.

And it is the ſame as having 2 Angles and a ſide oppoſite to one of them given, to find the reſt of the Triangle, as *prop.* 2, 3, 5, therefore by the Rule of Proportion, ſay, As the Angle C 90 *d.* is to *s.* AB the diſtance run, ſo is the Rumb or Angle A to the *s.* BC the Difference of Longitude, and ſo is the Complement of the Rumb to 90, *viz.* Angle B, to *s.* AC the difference of Latitude. In this Propoſition, as alſo in any other, where the Rumb is given, as *Prop.* 2, 4. having the Rumb given, and the Right Angle, you have alſo the third Angle given, being the Complement of the Rumb to 90, which makes it very eaſie to be reſolved. And thus you ſee theſe 6 Propoſitions of *Plain Sailing*, (or if there can be more Tranſmutations of theſe 4 principal Terms in *Navigation*) may all be reſolved by *Trigonometry*: So that hence you will clearly diſcern, that not only *Surveying*, but *Navigation* alſo is performed by the Reſolution of a Triangle, and that only a Plain Triangle; and Navigation (altogether) and *Surveying* for the moſt part, by a

Right

Right Angled Plain Triangle only: whereby the eaſineſs of both, and the excellency of *Trigonometry* do moſt manifeſtly appear.

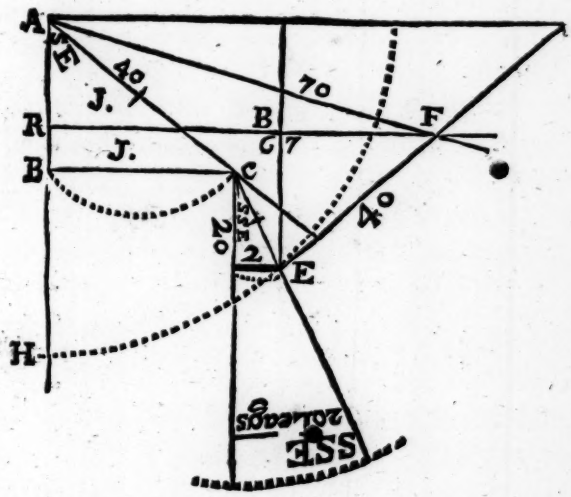
SECT. 6.

A Travis.

What hath hitherto been ſpoken concerning *Navigation*, reſpects only a direct ſingle Courſe where a man needs not Lavere, or change his courſe at all; and for a *Travis*, it is the ſame thing only wrought by ſo many more Triangles, every time you change your courſe conſtitutes a new Triangle, as by the following Example.

As ſuppoſe I ſet out from *Sommars Iſland*, lying in the Latitude of 32 *d.* 20 *m.* North Latitude, and Sail *S E* 40 Leagues; from thence I Sail *S S E*, till I alter my Latitude one Degree; and from thence upon ſome point between North and Eaſt I Sail 40 Leagues, till I alter my Latitude one Degree, 45 *m.* I demand what Latitude I am in; my diſtance from the place whence I ſet Sail: My departure from my firſt Meridian, likewiſe my Courſe made good: When all this is done and protracted, the difference of Latitude is AR, reckoning 20 Leagues, or 60 Miles to a Degree, is 21 Leagues, or 63 Miles; and therefore in degrees

grees, 1 *d.* 3 *m.* which becauſe it is Southerly, ſubſtract from your firſt Latitude, (if it had been Northerly, you muſt have added it thereunto.) And that is your preſent Latitude.



Secondly, The diſtance run (as if you had gone directly ſtreight) is AF 70 Leagues taken from the ſame Scale of equal parts, as AC or EF was taken.

Thirdly, The Departure from the Meridian is RF 67.

Fourthly, The point of the Compaſs that the Ship lies now from *Sommars Iſland*, is from H to F, *viz.* ESE half a Point Eaſterly.

PROBL.

PROBL. 1.

Sailing away *WSW*, I ſee a point of Land which I find to bear from me *WBN*, and having Sailed 6 *Leagues* further I find it bears from me *NWBW*, I would know how far it is Diſtant from me.

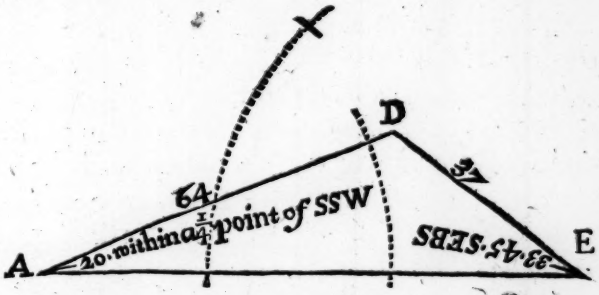
This is done in Plain Sailing after the ſame manner as to take an inacceſſible diſtance at 2 Stations at *Land*, as in *pag.* 197.

PROBL. 2.

A Merchant Man being in the *Latitude* of 43 *d.* falls into the hands of Pyrates, who amongſt other things take away his Sea-Compaſs, but when he is gotten clear, he ſails away as directly as he can, and after two days meets with a Man of War, who alſo had been the day before in the *Latitude* of 43 *d.* and had Sailed thence SEBS. 37 *Leagues*. He being deſirous to find thoſe Pyrates, the Merchant Man tells him he left them lying to and fro where they took him, and he had Sailed ſince at leaſt 64 *Leagues* between the South and Weſt, what Courſe ſhall the Man of War ſhape to find theſe Pirates.

This is done much after the ſame manner as the former, ſuppoſe AE the Parallel of 43 *d.*

43 *d.* D the place where the Ships meet, the Angle at E, the Rumb upon which the



Man of War Sailed SEBS 33 *d.* 45 *m.* DE his diſtance 37 *Leagues*, AD the diſtance Sail'd by the Merchant Man. Now by theſe you are to find the Angle A, which you will find 20 *d.* within ¼ point of *SSW*, to hath the Merchant Man ſailed, and therefore the Man of War muſt Sail *NNE*, within ¼ point of the Compaſs.

I might add many more ſuch like as you find them in *Norwoods* Epitomy, to which I refer the Reader; done after the manner of Plain Sailing, ſuppoſing the Degrees of *Longitude* and *Latitude* are both equal.

This *Travis*, as alſo the eight former Propoſitions are all wrought by that way of Sailing, which we call *Plain-Sailing*, or Sailing by the Plain Cart, which is very eaſie and alſo exact enough for ſhort Diſtances, and near the Equinoctial; in other Voyages it

it is ſubject to Error, as ſuppoſing the Degrees of Longitude to be the ſame in every parallel of Latitude; or that the Degrees both of Longitude and Latitude are the ſame all the world over, which is a grand Miſtake, as Mr. *Wright* makes it appear in his Errors of *Navigation*, and withal hath found a Remedy for it, by drawing a true Sea-Cart by the Meridian-Line, which he calls *Mercators Chart*, being according to *Mercators Projection*.

The Difference between Sailing by the *Plain Chart*, and Sailing by *Mercators Chart*; that is, the Difference betwixt keeping an account either by the one, or by the other, conſiſts in theſe particulars.

Firſt, In the *Plain Chart* both the Meridians and *Parallels*, or (in plain Termes) the Degrees of Longitude and Latitude, are all equal, and meaſured by the ſame Scale of equal Parts, and therefore by conſequence the Diſtance muſt be alſo meaſured by the ſame Scale, and ſo all the three ſides of any Plain Right-Angled Triangle repreſenting theſe, *viz.* The Baſe, the Perpendicular, and the Hypothenuſe are all meaſured by one and the ſame Line of equal Parts,

But finding this way, *viz.* by the *Plain-Chart* very erroneous, being grounded upon

errone-

erroneous principles, as is ſhewed by Mr. *Wright* in his errors of *Navigation*, as alſo by *Leyburn* in his ninth Geometrical Exerciſe. The Projection of the *Mercators Chart*, and keeping a reckoning at Sea by that, diſcovers the true Longitude, Latitude, and Diſtance moſt exactly. For the Meridians being great Circles of the Globe are alwaies equal, and the Degrees of *Latitude* meaſured thereupon are alſo equal:

But the Parallels (to the Equinoctial) are all leſſer Circles of the Globe, and towards either Pole alwaies grow leſſer and leſſer, (as may moſt evidently appear by the Globe it ſelf if you pleaſe to caſt your eye upon it:) And therefore the Degrees of Longitude which are meaſured upon theſe Parallels do differ in every Latitude: Therefore one ſide of *Mercators Chart* is the Meridian Line of unequal Parts, and the other ſide is a Line or Scale of equal Parts only: Upon the Meridian Line of the *Chart* is meaſured the Difference of Latitudes, and the Diſtance upon the Rumb. The Difference of Longitude is meaſured upon the Equinoctial Line, or Line of equal Parts: And in *Trigonometry*, becauſe the ſides of all *Triangles* ought to be brought to the ſame proportional parts one with another, that is the meaning of Meridional

nal Parts; for inſtead of the unequal Parts of the Meridian Line, you take the equal parts of the Line of Lines anſwering thereunto: Now how to find theſe Meridional Parts in reference to the Latitude you have plainly ſet down *pag.* 283. The ſame Rules will alſo ſerve for the Diſtance between any two places, provided that taking the ſame Diſtance in your Compaſſes, you ſet one Foot of your Compaſſes as much below the leſſer Latitude, as the other above the greater Latitude.

Then by the former Rules, *pag.* 283, find the Meridional Parts anſwering to the Degrees on the Meridian Line intercepted betwixt the feet of the Compaſſes, and thoſe are the Meridional Parts anſwering to the Diſtance upon the Rhumb, and to be uſed inſtead thereof.

So that you ſee in ſhort, the difference betwixt Sailing by the *Plain Chart*, and *Mercators Chart*, which differ in this, and this only: That in the Plain Chart both the difference of Longitude and Latitude, as alſo the diſtance are all meaſured by the ſame Scale of equal parts; but in *Mercators Chart* the difference of Longitude only is meaſured upon the ſcale of equal parts, and the difference of Latitude and diſtance muſt be alwaies meaſured upon, and taken out of the Meridian Line of the Chart: And when either the difference

of

of Latitude or the diſtance is oppoſed in Proportion to the Longitude, inſtead of the Degrees of the Meridian Line you muſt uſe the Meridional Parts anſwering thereunto, which by moſt are called the Meridional Diſtance, or Meridional difference of Latitude; but by Mr. *Leyburn* in his Geometrical Exerciſes, *pag.* 18. They are called the Proper diſtance, and the proper difference, which I ſhall retain in ſome of theſe following Problems taken out of the ſame Author.

SECT. 7.

In Gunters Sector *the Meridional Line being placed on the ſame ſide with the Line of Lines, you may thereby eaſily find theſe Meridional parts by theſe Rules.*

1. If you would know the difference of Latitude in Meridional parts betwixt two places, the one lying under the Equinoctial, (as the River of *Amazones*) the other in 50 degrees of North Latitude, ſuppoſe (the Lizard). Look for 50 on the Meridian Line, and right againſt it on the Line of Lines you find 57 *d.* 54 *m.* or in decimal parts of a degree, 57 *d.* $\frac{9}{10}$ and theſe are the Meridional parts required.

2. If both the places have Northerly or Southerly Latitude, as ſuppoſe one in 30 *d.* the other in 50 *d.* Extend your Compaſſes from

from one of the Latitudes to the other, as from 30 *d.* to 50 on the Meridian-Line. The ſame Extent will reach from the beginning of the Line of Lines to 26 *d.* 26 *m.* which is the Meridional parts required.

3. When one of the places hath South-Latitude, and the other North, as ſuppoſe 10 *d.* South, and the other 30 North. The Extent from the beginning of the Meridian Line to 10 degrees ſhall reach from 30 *d.* on the Meridian Line, to 41 *d.* 31 *m.* on the Line of Lines or equal parts, which are the Meridional Parts required.

Knowing then how to find theſe Meridional parts, Note that the difference of Latitude in theſe is called the Meridional difference of Latitude, and the true difference of Latitude is that which is found by ſubſtracting the leſſer Latitude out of the greater, the Remainder is the true Latitude. This being premiſed, I ſhall now proceed to theſe ſeveral Propoſitions.

PROBL. 1.

The difference of Latitude, and difference of Longitude being given or known, to find the Rumb.

As the difference in Latitude is to the diffe-

difference in Longitude, So is the Tangent of 45 to the Tangent of the Rumb.

Suppoſe the two places be the Lyzard in 50 degrees North, the other St. *Chriſtophers* in 15 *d.* 30 *m.* the proper difference of Latitude is $42\frac{12}{100}$, and the difference of Longitude 68 *d.* 50 *m.* Therefore the Extent on the Line of Numbers from $42\frac{12}{100}$ to 68 *d*, 50 *m.* ſhall reach the ſame way from the Tangent of 45, to the Tangent of 58 *d.* 26 *m.* which is the Rumb leading betwixt thoſe 2 places.

PROBL. 2.

The Difference of Latitude and the Rumb given, to find the difference of Longitude.

As Radius, or the Tangent of 45 to the Tangent of the Rumb, ſo is the proper difference of the Latitude to the difference of Longitude.

PROBL. 3.

By one Latitude, the difference of Longitude and the Rumb given, To find the other Latitude.

As the Radius is to the Co-Tangent of the Rumb from the Meridian, ſo is the

diffe-

difference of Longitude to the proper difference of Latitude.

PROBL. 4.

Having the Latitude of one place (50 *d.*) and the Rumb (33 *d.* 45 *m.*) leading from that place to another unknown; and the distance upon the Rumb from the first place to the second, (6 *d.*) To find the difference of Longitude ($5\frac{1}{2}$ *d.*)

As the Radius is to the sine of the Rumb from the Meridian, so is the proper distance upon the Rumb to the difference of Longitude.

PROBL. 5.

The difference of Longitude, one Latitude and the Rumb given, to find the distance.

As the Sine of the Rumb is to the difference of Longitude, so is the Radius to the proper distance.

PROBL. 6.

Difference of Longitude, and distance with the Latitude of one of the places given, to find the Rhumb that leads from one to the other.

As

As the proper distance is to the difference of Longitude, so is Radius to the Sine of the Rumb.

PROBL. 7.

The Longitude and Latitude of two places being given, to find the distance and Rumb.

As the proper difference of Latitude is to the Radius, so is the difference of Longitude to the Tangent of the Rumb from the Meridian: And as the Sine of the Rumb is to the difference of Longitude, so is the Radius to the proper distance.

PROBL. 8.

The difference of Latitudes and their distance upon the Rhumb given, to find the difference of Longitude.

As the proper distance is to Radius, so is the proper difference of Latitudes to the Co-sine of the Rumb from the Meridian: And so is the Sine of the Rhumb from the Meridian to the difference of *Longitude*.

PROBL. 9.

The Difference of *Longitude* of two places, their Distance upon the Rumb, and the

Latitude

Latitude of one of the places being given. To find the Difference of *Latitudes.*

As the proper distance is to the Radius, so is the difference of *Longitude* to the inclination of the Rumb to the Meridian: And so is Co-sine of the Rhumb to the difference of *Latitudes.*

PROBL. 10.

By one *Latitude*, Distance and Rumb, to find the other *Latitude.*

As Sine 90, or Radius is to the Cosine of the Rumb, so is the distance to the true difference of *Latitude*:

I shall now proceed to shew you Mr. *Philips*'s way of drawing a *Mercators Chart*, and how to measure both *Longitude*, *Latitude*, and *Distance* thereupon, and to keep an account of a Voyage thereby, as is performed by him in his Geometrical Seaman, which I recommend as a Book of very excellent use to Seamen and such as desire to be more at large informed in all the particulars concerning *Navigation* in all the three kinds: Of Sailing by the *Plain Chart*, *Mercators Chart*, or by the *Arch* of a great *Circle.* I shall in this place content my self only to Epitomize the Use and Figure of his *Mercators Chart* as follows.

Know-

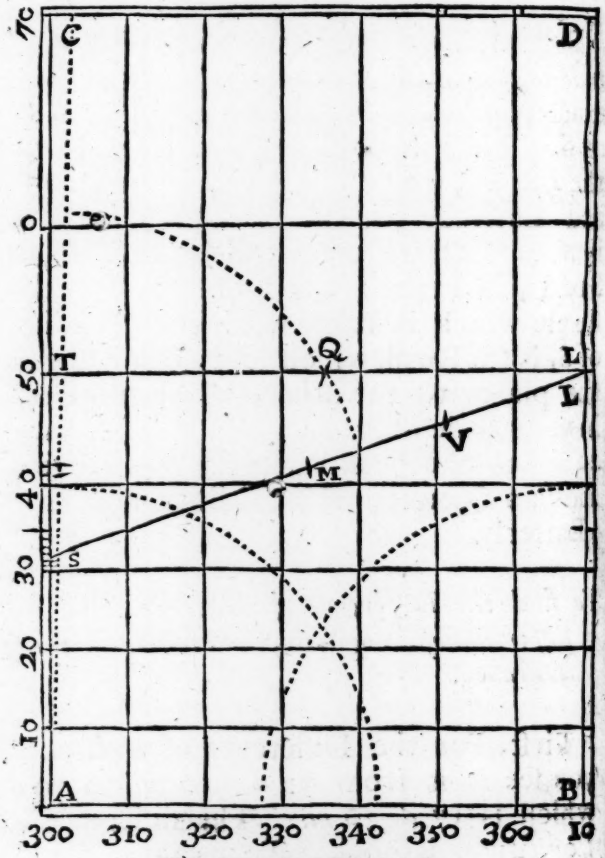
Knowing the Longitude and Latitude of two places, to ſet them upon the Chart, and to find the Rhumb leading from one of the places to the other.

Suppoſe *Summers* Iſland, whoſe *Longitude* is 300, and *Latitude* 32 *d.* 25 *m.* the exact place to ſet that down in the Cart is at *S.* Then ſuppoſe you would ſet down the *Lyzard*, whoſe Longitude ſuppoſe 10 *d.* and Latitude 50, the exact place for that is at L. Then draw a ſtreight Line from S to L, and that is the Diſtance; and the Angle which it makes with the Meridian of S, is the Rumb which leads from one of the places to the other, which meaſured in your Scale of Rumbs, you will find 71 *d.* 21 *m.* or the ſixth Rumb, and ſomething above a Quarter, which is ENE more than ¼ Eaſterly.

To find the Difference of Latitude, and the difference of Longitude, *and to Meaſure the Diſtance.*

Firſt, For the Difference of Latitude, you ſee it is from 32 *d.* 30 *m.* to 50 *d.* which is 17 *d.* 30 *m.* The difference of Longitude is 70 *d.* To meaſure the diſtance upon the Rumb betwixt S and L, do thus, Firſt, Divide the diſtance that is in

the Meridian Line between the two Latitudes into two equal parts, which in this Example will fall at 42 Degrees. Then



The Equinoctial Line.

take with your Compasses half the length of

of the Line LS which is LM, and with that distance set one foot of your Compasses in 42, the middle point betwixt the two Latitudes, you will find the other foot will reach downwards to 8 *d.* 30 *m.* and upwards to 63 *d.* 30 *m.* The Degrees in the Meridian Line betwixt these 2 points are 55 degrees, which is 1100 Leagues, and this is the true distance upon the Rhumb betwixt S and L.

To Measure a Parallel Distance between two places.

Suppose the places to be L and T both in the Latitude of 50 degrees, their difference of Longitude being 70, divide the distance betwixt L and T into 2 equal parts which is at Q. Take L Q in your Compasses, and setting one foot in 50 Degrees of the Meridian Line, or the degrees of Latitude, the other foot will reach downwards to 22 *d.* 30 *m.* and upwards to 67 *d.* 30 *m.* and if you count the degrees intercepted between these two points, you will find them 45 *d.* which multiplyed by 20, gives the distance between these two places 900 Leagues.

If two places differ only in Latitude and lye under the same Meridian: The Degrees between them upon the Meridian Line gives the Degrees, which multiplyed

by 20, bring them into Leagues, or by 60 into *Engliſh* Miles.

Having Sailed from the *Lyzard* about the Diſtance of 15 degrees upon the ſame Rumb as in the Chart, you would know what Longitude and Latitude you are come to, or the point where the ſhip then is.

Take 15 degrees in your Compaſſes from 50. (or L.) which ſignifies the Lyzard) out of the Meridian line; or the degrees of latitude whatſoever they be, which will reach to 35 *d.* in this Example.

Set this diſtance upon the Rumb line you Sail upon, which will be at V; lay your Ruler by that mark from 46 and 46 in the Meridian-line, and that is your latitude; do the like for your longitude, and you will find it 351, or thereabouts.

To avoid Sailing in a Parallel that is full Eaſt and Weſt at any time.

It will be your beſt way to ſteere no nearer to the Eaſt or Weſt than the 7 Rumb from the Meridian, for ſo you may by your obſervation of the latitude correct your account, which otherwiſe you cannot do, and in doing thus, you will not go much out of your way; for if the two places be diſtant 10 degrees, if you ſail one half of your way upon the ſeventh Rumb, you will raiſe the

Pole

Pole very near 1 degree, *viz.* $\frac{994}{1000}$, and if you lay the Pole as much upon the ſeventh Rumb the other way, ſo you will come to the place deſired, having run the ſame diſtance as before, *viz.* 5 leagues. Thus in a Voyage of 10 degrees, or 200 leagues, you go not fully 4 leagues out of your way, which is not one in 50, which will be well recompenſed, becauſe in Sailing thus upon the ſeventh Rumb, you may by the obſervation of the latitude correct your Account.

Note that a Travis may be drawn upon this Chart, as that in *pag.* 277 is done upon the Plain Chart, *mutatis mutandis* in the latitude and Diſtance.

And thus much ſhall ſuffice to have ſpoken of Sailing by the *Plain Chart*, or *Mercators Chart*. Thoſe that have a mind to underſtand the third kind of Sailing by the Arch of a great Circle, they may conſult *Philips* his Geometrical Seaman and other Authors: I ſhall not here inſiſt upon it, conceiving it to be rather a piece of Nicetie, than any great matter of ſolidity which I diſcern in it; yet withal do take it for an unqueſtionable verity, that it is the neareſt way by reaſon of the Sphærical Form of the Earth and Sea, and circularity of the Terreſtrial Globe. But to let this paſs, I ſhall conclude what I intend to ſay concerning *Na-*

vigation in theſe following particulars.

1. To ſhew how many leagues do anſwer to one degree of longitude in every ſeveral latitude.

2. To ſhew how many leagues do anſwer to a degree of latitude in every ſeveral Rumb.

3. To know the Burden of a Ship, and how to B ild a Ship to any proportion or Bigneſs.

1. To know how many leagues do anſwer to a degree of longitude in every ſeveral latitude.

As the Radius (or ſine of 90 *d.*) is to 20 in the line of Numbers, (or 60 if you deſire Miles) ſo is the Co-ſine of latitude (at *London* 38 *d.* 38.) to the number of leagues or Miles in that latitude (*viz.* at *London*) 37 $\frac{3}{10}$. Miles, or 12$\frac{1}{2}$ leagues, *fere.*

To do this by the Sector,

Take 20 leagues out of the line of lines, and make it a Parallel Radius by fitting it over in the ſines of 90 and 90, ſo his Parallel ſine taken out of the complement of the latitude, (*viz.* 25. 15.) will give the leagues (*viz.* 18.)

To do this another way:

Suppoſe you have a Quadrant, the one ſide whereof being the full Radius may be divided into 20 equal parts, the other ſide into 60;

60; then with your Compaſſes take the neareſt diſtance from the complement of the latitude to the ſide of the Quadrant, and that diſtance meaſured upon the line divided into 20 from the Center, will give the leagues upon the line of 60, which will give the miles anſwerable to one Degree of longitude in that latitude. Other waies there may be for doing this, but theſe are ſufficient.

2. To know how many leagues do anſwer to a degree of latitude in every ſeveral Rumb, or more Plainly to know how many leagues you are to ſail upon each Rhumb before you raiſe or depreſs the Pole one Degree.

As the complement of the Rhumb is to 20 leagues the meaſure of one degree at the Meridian, ſo is the Radius to the leagues anſwering to one degree upon the Rumb.

As ſuppoſe Sailing NEBN from 50 *d.* of latitude, it were required how many leagues the Ship ſhould run before it ſhould come to 51 *d.* of latitude: Becauſe this is the third Rumb, and the inclination thereof 33 *d.* 45 *m.* The Complement thereof is 56 *d.* 15 *m.* Therefore the Extent of the Compaſſes from 56 *d.* 15 *m.* upon the line of Sines to 20 upon the line of Numbers, the ſame extent the ſame way will reach from Radius or the Sine of 90 *d.* to 24, ſo many leagues muſt be ſailed to raiſe the Pole 1 *d.*

To do this by the Sector:

Take 20 leagues from the line of lines, and make it a parallel ſine of 56 *d.* 15 *m.* and his parallel Radius between 90 and 90 meaſured in the ſame line of lines, will ſhew you 24 for the Number of leagues required.

Sailing with the Ship.	Rhumb.	Leagues.	Do either Raiſe or Depreſs the Pole one Degree.
North or South	You keep the ſame Meridian		And 20 *Engliſh* Leagues.
Upon the 1ſt. Rumb	Having changed your Parallel a degree of Latitude, your departure from the Meridian is 4 Leagues.		& 20 1/2
Upon the 2	8 1/3		& 21 1/3
3	13 1/3		& 24
4	20		& 28 1/3
5	30		& 36
6	48 1/3		& 52
7	101		& 102 2/3
East or Weſt	You are ſtill in a Parallel, and neither raiſe, nor lay the Pole at all.		

To Convert Degrees and Minutes into Hours and Minutes.

Note that 15 Degrees makes an Hour, therefore divide the degrees by 15, the Quotient will be Hours: The Fraction (if any

any remain) multiply by 4, (becauſe every Degree is 4 Minutes of time) and to the Product add every 15 Minute of a Degree (if any be) for ſo many makes a Minute in time. This will exactly reduce Degrees and Minutes to Hours and Minutes. As for Example 118 *d.* by 15. the Quotient is 7 Fraction 13, which multiplyed by 4, is 52. *Total* 7 H. 52 M. as above.

But for your ready finding of your true Northing and *S*outhing, or Eaſting and Weſting, or in other Terms your Difference of Latitude and difference of Longitude by the Rumb and diſtance given, or by knowing how many Leagues you have ſailed upon any Rumb: You have excellent Tables in *Philips*'s Geometrical Sea-man, and in *Sturmy*, or to ſupply thoſe you may have a Traverſe Scale, which is a Sliding Scale, on one ſide having a Line of Numbers, on the other ſide the Rumbs: The uſe of it is thus; Set your diſtance run againſt the Radius (which is the eighth Rumb) and the Number of Leagues againſt the Rumb you ſailed upon, is the difference of Longitude, or your departure from the Meridian, or your Eaſting or Weſting (call it which you will) and the Number of Leagues oppoſite to the Complement of the Rumb is the difference of Latitude, or your Northing or Southing: As for Exam-

ple, ſuppoſe you Sail 57 Leagues upon the third Rumb, and would know how many Leagues you have altered your longitude or latitude: Set 57 the leagues ſailed to Radius (or the eighth Rumb;) look againſt the third Rumb, there you find 31 $\frac{7}{10}$ leagues for your departure from the Meridian or difference of Longitude, and againſt the complement of the ſaid Rumb, which is 5, or the fifth Rumb, there you will find 47 $\frac{4}{10}$ leagues, and ſo many leagues is your difference of Latitude: The like of all other.

The Uſe of the Sea Quadrant.

This Inſtrument conſiſts of 3 Vanes and 2 Arches, one of 60, and the other 30, which together make 90 Degrees, and therefore call'd a Quadrant: The 3 Vanes are the ſight Vane upon the 30 Arch, the Shadow-Vane upon the 60 Arch, and the Horizon-Vane at the Angle A: So turn your back of the Sun, and move the ſights, till the ſhadow and the Horizon be ſeen together, and then upon the 60 or leſſer Arch obſerve the Degrees cut by the upper edge of your Shadow-Vane, and add thoſe to the degrees cut by the inſide of the Sight-Vane upon the 30 Arch, which Degrees added together, the ſumm is the Complement of the Altitude, or the Suns diſtance from the

To keep a reckoning upon a Plat of a whole Voyage.

Suppoſe you Sail from the *Lizard* South-Eaſtward (which we ſuppoſe in the Latitude 50°, 00 degrees) and the difference of latitude 3 degrees, which deduct from 50 *d.* and then the latitude the ſhip is now in is 47 *d*, and ſuppoſe your courſe run 50 leagues Eaſtward.

Therefore to ſet the place of your ſhip upon your Plat, you muſt uſe two pair of Compaſſes, with one pair take the extent between the latitude 50 degrees and 47 degrees, and with that extent ſet one point of your Compaſſes in the *Lizard*, the other extend towards the place where the Ship is now, but ſo that your Compaſſes ſtand parallel to a North and South Line. This done, keep one foot of your Compaſſes in that point, and with the other pair take the departure 50 leagues from the Scale of leagues, then interchange your Compaſſes, placing this laſt pair in the point where the other pair ſtood, ſet this departure 50 leagues to the Eaſtward; but ſo that your Compaſſes may ſtand parallel, and to an Eaſt and Weſt Line. (This you may find by trying with your other Compaſſes whether the legs be æquidiſtant from the next Eaſt and Weſt line.) This

This ſecond point thus found upon the Plat is the place of the Ship (according to your reckoning) which was required. This I have ſeen practiſed by ſeveral Ship-Maſters upon Maps, and is a very ready way, marking the place where the Ship is with Chalk, which may be rubbed out again when you come at the next Station.

How to know what Burthen a Ship is of.

The general receiv'd Opinion is to take the length at the Keel, the Breadth at the Beam, and his Depth in Hold, which multiply one in the other, and divide the laſt product by 100, gives you his Tunnage, which is the Kings allowance: But Merchants divide by 110; but you muſt note this is of ſuch Ships as are built Taper-wiſe from the Keele, but others think 95 a number reaſonable.

Length 75 foot, Breadth 29, Depth 13 } Product is 28275, which divided by 100, gives 282¾ Tunns.

Suppoſing a Ship of 100 *Tun, and you would make another Ship of double Burthen.*

Suppoſe this Ship of 100 Tun be 44 foot long by the Keel, 20 foot broad midſhip Beam,

Beame, 9 foot deep in Hold. Multiply each of theſe Numbers cubically; double the Product, and then extract the Cube Root. As for Example, Multiply 44 by 44, makes 1936, and that again by 44 makes 85184. for the Cube number, which being doubled (becauſe the Ship is to be double the burthen of the other) ſo it makes 170368: Extract the Cubique Root of this Number, and it will yield 55 feet $\frac{43}{100}\frac{7}{0}$ parts, which is 5 inches, and almoſt $\frac{1}{4}$ inch, and ſo you muſt do by all the other dimenſions of the Ship to find them, and ſo make them proportionable.

What due Proportion ought to be uſed in the Building of all Ships whatſoever.

The due proportion of a Ship is, That the longitude of the Veſſel whatſoever it be more or leſs ought to be divided into 300 equal parts; of the which parts 30 muſt be aſſigned to the Depth, and the Breadth ſhall contain 50, or the $\frac{1}{6}$ part of the longitude, ſo ſhall the Ship be both proportionable, and more ſafe for Traffick.

To conclude pleaſantly, I ſhall here give you Biſhop *Wilkins* device in his *Mathematical Magick* of making a Ship to Sail under Water: The Difficulties herein he reſolves thus.

1. No

1. No land Creatures (eſpecially men) can live without Air wherein they may breathe, and therefore ſuch a Veſſel muſt be very large and ſpacious; ſo as to allow Refrigeration.

2. If ſo, How is it poſſible that they ſhould ſwim or ſail along. To this he ſaies, That thoſe Bodies which are carried on the Water, be they never ſo big or ponderous, (ſuppoſe equal to a City or whole Iſland) yet they will alwaies ſwim on the top, if they be but any thing lighter than ſo much Water as is equal to them in bigneſs.

3. How to poiſe this Veſſel ſo equally, That it ſhall neither riſe too high, nor ſink too low: This he propounds to be done by hanging more or leſs great weights at the bottom.

4. How the motion of this Veſſel muſt be made, which muſt be by Oars put thorough the ſides of the Ships with leather-bags made faſt to the Hole, and tyed cloſe to the Oars, by which means the Oars may move, and yet let in no Water at all.

5. The ſame device doth remedie the 5 and laſt, and greateſt difficulty, which is to take any thing into the Veſſel, or put any thing out of the Veſſel without admiſſion of any Water: How this may be done, he propounds thus; Suppoſe a leather-bag of what bigneſs you pleaſe made faſt

and

and close to a Window round about, being also ty'd close about towards the Window, then any thing that is to be sent out may be safely put into that end within the Ship, which being again close shut or tyed, and the other end looſed, the thing may be safely sent out: And by the like means how to receive any thing into the Ship or Vessel is easie to conceive.

But I shall sail no further in this Discourse, the Voyage in such a Vessel being near as difficult and dangerous to perform, as to fly into his *New World of the Moon*, though he tells you *pag.* 219. That if a Body were above the Sphere of the Magnetical Vertue of the Earth (which he supposes 20 Miles upwards, or so) he might there stand as firmly in the open Air, as he can now upon the Ground.

Or Secondly, If this Sailing under Water could be performed, it would not be of much usefulness; therefore shall rank this amongst those other conceits which are demonstrable, but not practicable, such as pulling up Trees by the Roots with a single hair, or by the breath of a mans mouth: Or *Pythagoras* writing in the Moon by reflection: Or *Archimedes* his moving *datum pondus cum data potentia*; that if he did but know where to stand and fasten his Instrument, he could move the whole Terrestrial Globe.

Globe. More uſeful (if at all practicable) is his perpetual motion; but both uſeful and practicable, and in many places practic'd and us'd is his Sailing Chariot, like that in *Holland*, which with the Wind would paſs from *Scheveling* to *Putten* in two hours, being diſtant more than 42 *Engliſh* Miles.

But paying due Honour and reſpect to the mention of this Reverend, Learned, and Ingenious Author, I ſhall here conclude what I have to ſay of *Geometry* in general, and in particular as to *Surveying* and *Navigation*; and proceed according to my propounded Method to more ſolid and ſerious *Mathematical Propoſitions* and *Practical Concluſions in Aſtronomy*.

THE

The Third BOOK.

OF ASTRONOMY.

CHAP. I.

HAving hitherto treated of *Arithmetick* in the four kinds of it, *Common*, *Decimal*, *Logarithmetical*, and *Inſtrumental*, and alſo of *Geometry* both general in the Reſolution of Geometrical Problems and Propoſitions; and in the producing, reducing, and meaſuring the three principal Geometrical Figures: And likewiſe of *Geometry* in ſpecial applyed to *Surveying* and *Navigation*: I come now in the third place to treat of the third *Mathematical Liberal Science*, which is *Aſtronomy*: And as *Geometry* may be ſaid to meaſure the Earth, or this whole Terreſtrial Globe of Sea and Land, by taking Heights and Diſtances

ſtances at Land in *Surveying*, and by taking Heights and Diſtances at Sea in *Navigation*: So *Aſtronomy* may be ſaid to meaſure the Heavens by taking the Heights and Diſtances of thoſe Heavenly Bodies, *viz.* The Planets and Fixed Stars.

But that theſe Heights and Diſtances here principally meant and intended, may be entirely handled, and that other things which concern *Aſtronomy*, may be the better underſtood, I ſhall premiſe theſe ſeveral particulars, and proceed in this Method.

1. I ſhall give you the number and names of theſe Heavenly Bodies, *viz.* the Planets and Fixed Stars.

2. I ſhall ſhew you their Order, both in the Ptolemaick and Copernican *Syſtems* of the World, and give you the Reaſons given on both ſides, *pro & con* for each Syſteme.

3. I ſhall ſhew you their Periodical Revolutions.

4 I ſhall ſhew you the Magnitudes.

5. Their Heights or Diſtances from the Earth:

Which will lead me by the hand as it were in the laſt place to ſpeak of theſe other Heights and Diſtances which I make the Sum and Subſtance of *Aſtronomy* as it ſtands in Analogy with *Geometry* in this place: And theſe, to wit, the latter ſort of Heights and

and Distances are properly Questions of the Sphear, therefore I shall explain to you the *Projection of the Sphere in plano*, and the *Resolution of Spherical Triangles* grounded thereupon; by which *Spherical Triangles* these *Astronomical* Questions are resolv'd; as *Surveying* and *Navigation* are by *Plain Triangles*. But to proceed according to my proposed Method.

CHAP. II.

To give you the Number and Names of the Planets and Fixed Stars.

1. AS for the Planets, all agree as to the Number that there are 7, but which these seven are there is some difference; *Aristotle*, *Ptolomy*, and the rest of the *Peripateticks* do number them thus, *Saturn*, *Jupiter*, *Mars*, *Sol*, *Venus*, *Mercury*, *Luna*; making the Earth the Center of the World: But *Pythagoras*, *Copernicus*, and the Ancient *Pythagoreans* do make the Sun the Center of the World, and this Earth to be a Planet instead of the Sun, and to move in the same Orb which is ascribed to the Sun by *Ptolomy*, and so numbers the Planets thus, *Saturn*, *Jupiter*, *Mars*, *Tellus*, *Luna*, *Venus*, *Mercury*, and *Sol* the Center, and

and ſome will not have the Moon to be a primary Planet, but only a ſecondary, and that all the reſt of the Planets as well as this earth have their Moons to move about them, and for *Jupiter* there are 4 obſerved to move about him, which they call *Satellites*; and *Saturn* alſo is obſerved to have his concomitants.

Secondly, For the number of the Fixed Stars, they are vulgarly and commonly conceived to be innumerable, yet *Mathematicians* do ſay they may be numbred, and do take notice only of 1025, or at moſt of 1161, comprized in 48 Conſtellations,

	21	In the Northern Hemiſphere.
	15	In the Southern.
	12	In the Zodiack, or the Zodietical Conſtellations, being no other than the 12 Signs.
In all	48	

The

The Names of the *Northern* Conſtellations are

	Containing Stars
1 Urſa Minor	7
That in the end of the Tail is called the Pole Star.	
2 Urſa Major.	27
3 Draco.	31
4 Cephus.	11
5 Bootes.	22
6 Corona Borea	8
7 Engonaſis, *or* Hercules	29
8 Lyra.	10
9 Olor, *or* Cignus.	17
10 Caſſiopeia	13
11 Perſeus	26
12 Auriga	14
12 Conſtellations	215 Stars.

	Containing Stars
13 Ophiucus, *or* Serpentarius.	24
14 *S*erpens	18
15 Sagitta, *or* the Arrow	5
16 Aquila, *or* Vultur volans	9
17 Delphinus the Dolphin	13
18 Equiculus, *or* little Horſe.	4
19 Pegaſus, *or* the winged Horſe.	20
20 Andromeda	23
21 Triangulum the Triangle	4
9 Conſtellations Stars	117
12	215
21 Northern	332

The Names of the 15 Southern Conſtellations.

N°.			N°.		
1	Cetus the Whale	22	9	Crater the Cup	7
2	Fluvius Eridanus	34	10	Corvus the Crow	7
3	Lepus the Hare	12	11	Centaurus	37
4	Orion	38	12	Lupus the Wolf	19
5	Canis Major	18	13	Ara the Altar	7
6	Canis Minor	2	14	Corona Auſtrina the South Garland.	13
7	Argo Navis	41	15	Piſcis Auſtrinus	12
8	Hydra	25			
8 Conſtellations. Stars		192	7 Conſtellations. Stars		102
			8		192
			15 South Conſtellat.		294

The Names of the 12 Zodiatical Conſtellations.

N°.			N°.		
1	Aries the Ram	13	7	Libra	8
2	Taurus	23	8	Scorpio	21
3	Gemini	18	9	Sagittarius	31
4	Cancer	9	10	Capricornus	28
5	Leo	27	11	Aquarius	42
6	Virgo	26	12	Piſcis	24
6 Conſtellations. Stars		116	6 Conſtellations. Stars		154
			6		116
			12 Conſtellations.		270

Theſe

These are the 48 Constellations taken out of Mr. *Moxons* Book *De Globis*, where you may find their several Poetical stories, which are not unpleasant to read: And thus much shall serve for the Number and Names both of the Planets and Fixed Stars.

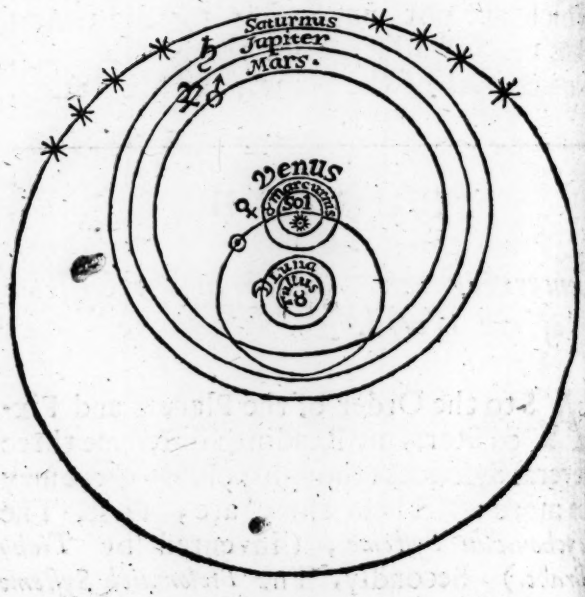
CHAP. III.

Concerning their Order in the Systeme of the World.

AS to the Order of the Planets and Fixed Stars, my Reading affords me three several Systemes (though possibly there may be more:) These three are, First, The *Tychonican Systeme*, (invented by *Tycho Brahe.*) Secondly, The *Ptolomaick Systeme* (invented and maintained by *Ptolomy* and the *Peripateticks.*) Thirdly, The *Copernican Systeme*, being the Ancient Opinion of *Pythagoras* and his followers, but revived by *Copernicus*, and since imbraced by most Modern Mathematicians, though Vulgar Capacities do take it for a strange thing, *monstrum Horrendum, &c.* I shall give you these three several *Systems* in order, and first, The *Tychonican.*

This

The *Tychonican Systeme.*

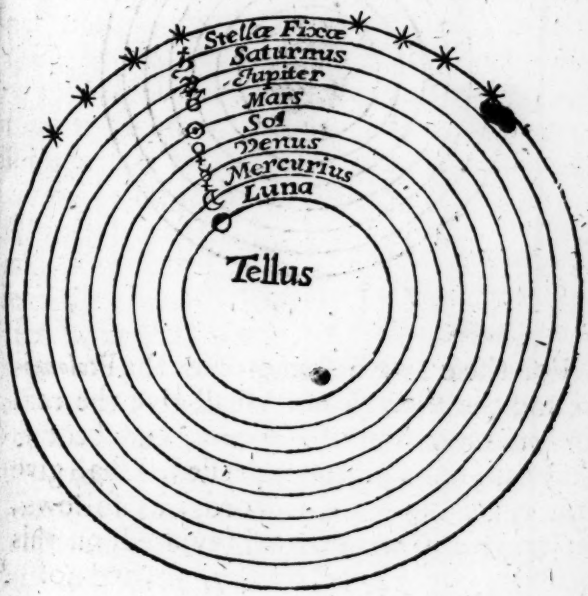


This Systeme was Invented by the Illustrious *Ticho Brahe* a Nobleman of *Denmark*, Lord of *Knudthorp* in the Island of *Shonen*, not far from *Elsenbourgh*, who framed this Hypothesis as a mean betwixt *Ptolomy* and *Copernicus.*

By this he supposeth that *Mercury*, *Venus*, and all the other Planets (except the Moon) in their motion, respect the Sun as their Center: So that *Saturn* in opposition to the Sun

Sun is nearer the Earth than *Venus* in Apogæon, and that *Mars* in oppoſition to the Sun, is nearer to the Earth than the Sun it ſelf, as may appear by Inſpection from the Hypotheſis it ſelf.

The *Ptolomaick Syſteme.*



The *Copernican Syſteme.*

For theſe two Syſtemes, *viz.* the *Ptolomaick* and the *Copernican*, I ſhall give the reaſons *pro & con* out of *Varenius*, and becauſe they cannot be better expreſſed, I ſhall give you them in his own words, as follows, *Geographia Generalis* abbreviated on this manner, *pag.* 34, 35, 36, 37, 39, and 40.

Rationes Copernicani *Syſtematis.*

1. Quoniam tantus eſt numerus ſtellarum quæ horis 24 circa Tellurem circuitum videntur perficere, & hæc apparentia per unius

unius Telluris (manentis in ſuo loco) motum explicari poteſt, ideo magis rationi conſentaneum eſt hunc ſtatuere quam illum; ſicut nobis in Navi ſedentibus, & appropinquantibus ad ſtationem multarum Navium quæ apparent nobis appropinquare, non ideo hiſce motum aſcribimus; & omnino cum natura non ſoleat facere per plura quod per pauca poteſt, veriſimile eſt, in hac quoq; re illud obſervatum eſſe.

2. Quia incredibilis & omnem cogitationem ſuperans celeritas motus ſtellarum illius foret: etenim cùm infinito ferè ſpatio a Tellure abſint, & vaſtiſſimus orbis illis percurrendus ſit, uno horæ minuto ad minimum per Centena millia milliarium ferri deberent. Contra, Si Telluri motus aſcribatur hic primus, manet hæc in ſuo loco, neq; de minima celeritate timendum eſt, quia Tellus circa ſuum axem rotatur.

3. Accedit argumento huic majus Robur ſi comparemus vaſtitatem corporum cœleſtium cum terreſtri. Etenim cum Sol ad minimum ducenties major ſit Tellure, ſtellæ vero fixæ vel millies majores, cui non veriſimilius fiat Tellurem rotari circa ſuum Axem motu naturali, quam tanta corpora incredibili pernicitate de loco in locum moveri?

4. Quoniam omnes Aſtronomi illuſtriores cum Tychone coacti apparentiis jam

negant Orbes ſolidos; quibus antiqui ad faciliorem ſtellarum motus Hypotheſin utebantur. Ideo multo incredibilior videtur illarum circumlatio circa Tellurem. Negant autem Orbes ſolidos, quia ſi hi eſſent, concedenda eſſet penetratio orbium mutua, cum quidam planetæ in alterius alicujus Sphærâ deprehendantur frequenter.

5. Nulla Ratio reddi poteſt cur Stellæ circa Tellurem moveantur, cùm contra cur Terra & reliqui planetæ circa ſolem cum reliquis planetis moveantur, aliqua poſſit dari Ratio.

6. Quia nec Polus, nec Axis eſt realis circa quem Stellæ ferri ponuntur; contra in Tellure & polus & Axis eſt.

7. Quia multo facilior eſt Navigatio ab occidente in orientem, quam ab oriente in occidentem. Etenim ex *Europa* in *Indiam* navigatur menſibus circiter quatuor, ex *India* in *Europam* reditur ſex menſium ſpatio circiter. Nimirum quia in illa Navigatione in eandem plagam cum Tellure moventur, in hac vero in contrariam.

8. Quia omnes apparentiæ cæleſtes, ortus, occaſus ſyderum, dierum incrementum, *&c.* poſſunt explicari, ſi Tellurem moveri ponamus.

Imprimis autem hujus Hypotheſis commoditas & neceſſitas conſpicitur in admirandis illis Planetarum affectionibus ad quas explicandas

eandas *Ptolemaici* multos circulos Epicyclos & eccentricos ſine ulla ratione excogitare coguntur: *Copernicani* autem illas ex Telluris motu ſecundo circa ſolem facili negotio deducunt ita, ut manifeſtam illarum cauſam reddant, & adeo facilem ut vel indocti illam capere poſſint, Nimirum.

1. Cur Planetæ interdum retrogradi videantur, & quidem *Saturnus* ſæpiùs & diutiùs quam *Jupiter*, *Jupiter* quam *Mars*, *&c.* interdum celeriori motu ferri, interdum ſtationarii eſſe.

2. Cur *Venus* & *Mercurius* nunquam poſſint tota nocte conſpicui eſſe.

3. Cur *Venus* nunquam majori a ſole intervallo, quam ſexaginta graduum, *Mercurius* non majori, quam triginta graduum intervallo diſcedat, & ideo nunquam oppoſiti conſpiciantur ſoli.

4. Cur *Venus* ejuſdem dici & veſpere poſt ſolem & mane ante ſolem poſſit conſpici.

Plures apparentias afferre ſuperſedeo: ſed illæ præcipue ſunt quæ per hunc motum Telluris adeo appoſite & jucunde explicentur, ut admirandum potius eſſet ſi Tellus non moveretur talibus apparentibus Phænomenis.

9. Sol non tantum fons lucis eſt, quæ tanquam clariſſima fax illuminat Tellurem, Lunam, Venerem & reliquos ſine dubio planetas, ſed etiam focus caloris & vitalis

ſpiritus quo totum hoc univerſum foveri & ſuſtentari videtur. Ideo medium locum omnium obtinere, & hos circa eum moveri probabile.

10. Sole collocato in medio redditur aliqua cauſa quare reliqui planetæ & Tellus circa eum ferantur, nimirum quia ſol vaſtiſſimum corpus eſt, & magnis viribus præditum, ideo reliquos planetas ad motum excitat.

11. Solem circa Axem ſuum rotari probant obſervationes Galilæi & Scheineri de maculis ſolaribus. Hac igitur ratione reliquis planetis circumeundi cauſa exiſtit, nec videtur ei alius motus attribuendus.

12. Si Telluri locum inter Martem & venerem, ſoli vero centrum attribuamus, apte reſpondet motus ſingulorum planetarum diſtantiæ a Centro, quod in Ptolemaica Hypotheſi non fieri patet ex motuum ſolis, Veneris & Mercurii conſideratione.

Verum enim vero *Ariſtotelici* cum *Ptolemaicis* ſententiam *Pythagoricorum* pluribus impugnat Argumentis & ſuam Hypotheſin probare conantur hiſce rationibus.

Rationes Ptolemaici *Syſtematis ſequuntur.*

1. Terram eſſe ineptam ad motum propter gravitatem; & Gravia feruntur ad Centrum mundi, Tellus autem graviſſimum corpus

corpus est, itaq; centrum illud occupat.

2. Gravia a Tellure discederent versus centrum universi, nisi in Tellure hoc centrum esset.

3. Partes Telluris moveri naturaliter motu recto ad centrum, ergo circularem motum esse contra naturam ejus.

4. Si Tellus moveretur, lapidem e turri demissum non posse cadere ad pedem turris, vel globum è tormento explosum versus orientem ad metam aliquam (vel etiam si avis quædam ad orientem volaret) non posse hanc attingere, si meta cum tota tellure versus orientem moveretur, vel saltem celeriorem fore attactum, si ad occidentem emissus esset globus.

5. Neq; turres neq; ædificia consistere posse, sed propter illum Telluris motum collapsura esse, neq; homines à vertigine immunes fore.

6. Quia videmus (inquiunt) stellas mutare locum, non autem Tellurem.

7. Quia Tellus est in centro mundi: centrum autem non movetur.

8. Quia sacræ literæ stabilitatem Telluris confirmant.

Ad hæc Argumenta Copernicani *respondere solent hunc in modum.*

1. Ad primum negando totam Tellurem

gravem esse; namq; gravitas est partium ad totum Homogeneum tendentia: Et talis gravitas quoq; in lunæ partibus & solis.

2. Et aliorum gravium motus non ad Centrum universi sed ad Corpus Homogeneum est, ut ex partibus Lunæ, Solis, magnetis probatur.

3. Motum illum rectum esse partium Telluris, non totius Telluris, atq; hujus circularem motum non impedire illarum rectilineam Lationem, quod declaratur a partibus Lunæ & Solis.

4. Ad hoc Argumentum respondetur triplici modo 1: Primo enim gravia talia non ad Centrum primario feruntur, sed ad ipsam tellurem, & ideo brevissima linea ad superficiem ejus; brevissima autem est hæc quæ turri respondet: Sicut ferrum non ad centrum magnetis, sed ad magnetem tendit. 2. Totus aer adhæret telluri & cum hac movetur, ideo etiam talia gravia demissa simul hunc circularem motum acquirunt & moventur tanquam in vase. 3. Gassendus crebrâ experientiâ demonstravit, quod si ex moto corpore aliquid projiciatur, hoc Projectum etiam illo motu corporis moveri, exempli gratiâ lapidem dejectum è fastigio mali navis celerrime motæ, tamen non relinqui à nave, sed ad pedem mali decidere: Et è pede mali explosum perpendiculariter è sclopeto globum, rursus perpendiculariter

decidere

decidere. Itaque allata objectio nihil valet: *Or if a Man jump in a Ship, he will not be able to paſs further, whether he jumps with, or againſt the motion of the Ship.*

5. Ad quintum Argumentum dicimus tale quid locum non habere: quia motus eſt æquabilis, nec in aliud corpus impinget, & ædificia tanquam corpora gravia & Telluri homogenea moventur tanquam in navi. Etenim in navi celerrimè etiam vel tardiſſimè mota, ſi modo æquabilis ſit navigatio hoc eſt ſine fluctibus, & aquâ plana, deprehendimus erecta corpora non everti, imo pocula vino plena nihil effundere.

6. Ad ſextum, dicimus mutationem loci ſtellarum non ſentiri, ſed ſitus reſpectu noſtri mutationem deprehendimus: poteſt autem hæc ſitus mutatio animadverti & eſſe, ſive nos cum Tellure ſive nobis immotis ſtellæ moveantur, vel etiam & nos & ſtellæ.

7. In ſeptima objectione & major & minor falſa eſt, vel ſaltem dubia.

8. Ad Octavum reſpondetur. 1. Sacram ſcripturam in rebus Phyſicis Loqui ſecundum apparentiam & vulgi captum, exempli gratia quando Luna cum Sole dicitur magnum luminare quod ad noctem illuminandum creatum ſit, cum tamen nec Luna magna ſit reſpectu ſtellarum & Telluris, nec proprium lumen habeat, neq; omnibus noctibus

ſibus illuminat terram. Ita ſolem ab extremitate ire, & ad extremitatem redire dicit ſcriptura, cum tamen revera talis extremitas nulla ſit. Ita in *Jobi* libro attribuitur Telluri figura plana & quadrata, cui columnæ ſint ſuppoſitæ, quibus innitatur, quod quidem haudquaquam ita intelligendum eſſe, vel vulgus novit. Plura loca adduci poſſint: ſed ſufficiunt hæc; namq; ſacræ literæ nobis non ad philoſophandum ſed ad pietatem colendam conceſſæ ſunt. Secundó Loca quædam ſcripturæ adduci ſolent, quæ non de immobilitate ejus, ſed de conſtantia & duratione loquuntur, ut locus ille quem e *Jobo* attulimus.

Thus you have the Reaſons both *pro & con* laid down by *Varenius*, both for the *Ptolemaick* and *Copernican Syſtemes* in reference to the order wherein theſe Planets and Fixed Stars are placed: I ſhall leave it to every one to be of what Opinion he ſees the moſt reaſon for: But however cannot paſs by what I find in that great Mathematician Biſhop *Wilkins* after that he hath argued ſeriouſly for the *Copernican*; he adds jocantly, That they that would have the Earth ſtand ſtill, and the Sun to move, are like the Cook who would not roſt his Meat by turning the meat about to the Fire, but rather by turning the Fire round about the Meat. (*World in the Moon.*

If

If the Reader deſire to know more concerning theſe matters, let him read the foreſaid Author, and *Vincent Wing* his *Harmonicon cœleſte*, and ſuch as have treated largely of theſe matters. Theſe may ſuffice for my preſent purpoſe, and ſo ſhall paſs on to the Third particular propounded to be conſidered, which is the Motion or Periodical Revolution of theſe Heavenly Bodies.

CHAP. IV.

Concerning the Motions and Periodical Revolutions of the Planets and Fixed Stars.

ACcording to the two different Hypotheſis of *Ptolomy* and *Copernicus*, there are you may ſee two different Centers to their ſeveral Syſtemes, *viz.* the Earth and the Sun, the one ſtanding ſtill, the other is ſuppoſed to move in its proper Orb betwixt *Venus* and *Mars*: If then according to *Ptolomy* the Earth be the Center, then muſt the Sun move, and alſo the Fixed Stars, or the Starry Firmament.

1. The Sun muſt move about the Earth in 24 hours, making Day and Night, which according

according to its proper Orbes Diſtance from the Earth of 1142 Semidiameters muſt be above 7570 *Engliſh* Miles in a Minute of time, and finiſheth his periodical Revolution thorough each ſide in the Zodiack, call'd his ſecond motion in a year, or 365 days 6 hours, *feré*, *Wing* 365 days 5 hours 48 Minutes, 55 Seconds, by this making Winter and Summer, Spring and Autumn.

2. The Earth ſuppoſed the Center, the Starry Firmament muſt alſo move round about the Earth in 24 hours, and the motion of the Starry Firmament being proportionable to its diſtance from the Earth, a Star in the Æquator muſt move 12598666 *Engliſh* Miles in an hour, or 209974 in a Minute, ſo that if an Horſeman ſhould ride every day 40 Miles, he could not ride ſuch a Compaſs in a 1000 years as the Starry Firmament moves in an hour, which is more than if one ſhould move about the Earth 1000 times in an hour, and quicker than poſſible can be imagin'd; and if a Star ſhould fly in the Air about the Earth with ſuch a prodigious quickneſs, it would burn and conſume all the World here below: This made *Copernicus* not unadviſedly to attribute this motion to the Earth, and that the Orb of the Fixed Stars ſtands ſtill, as well as the Stars in the ſaid Orb are fixed in their places and due diſtances from

from one another. Suppoſing then with *Copernicus* the Sun to be the Center of the Univerſe; (but allowing it withal a circumrotation upon its own Axis) I ſay, ſuppoſing the Sun the Center, there muſt be aſcribed to the Earth at leaſt a two-fold motion: The firſt is its Diurnal motion, anſwerable to the *Ptolomaick* Diurnal motion of the Sun and Stars: This performed upon its own Axis in 24 hours, and conſtitutes day and night. The ſecond motion of the Earth is his motion *a loco in locum* in its proper Orb betwixt *Venus* and *Mars*, as you may ſee in the Syſteme, and this is performed in a year, or 365 days, 6 hours, *fere*; or 365 *d.* 5 *h.* 48′. 55″. inſtead of the periodical revolution of the Sun according to *Ptolomy*, and this conſtitutes the 4 ſeaſons of the year, Spring, Summer, Autumne, and Winter; as is excellently demonſtrated in Biſhop *Wilkins*'s *World in the Moon.*

Now as to the former of theſe motions, or *ſecundum primum Telluris motum* upon its own Axis; you muſt know that every place upon the Terreſtrial Globe (as you may plainly ſee by turning about an Artificial one) doth not move alike, but places under the Equinoctial do move the faſteſt or quickeſt, and nearer either of the Poles they move ſlower, in ſuch proportion as the

the Degrees in each parallel of Latitude are greater or leſs: Therefore to know how many Miles in an hour any particular place is carried by this motion, take this proportion or Analogy. As the Radius is to the Co-ſine of the Latitude; ſo is 15 *d.* or 900 *Engliſh* Miles to the Number of Degrees or Miles required.

As for Example, *Amſterdam* is in the Latitude of 52 *d.* 23 *m.* its complement, or co-ſine is 37 *d.* 37 *m.* ſo that as 90, or Radius is to 37 *d.* 37 *m.* ſo is 900 to 548 *Engliſh* miles; or ſo is 225 *German* Miles, (4 *Engliſh* miles making one *German*) to 137 *German* Miles; or ſo is 15 degrees to about 9 degrees, which *Amſterdam* moves in an hour, or 9 of theſe *German* miles in 4 minutes of time: Now how theſe Numbers 15, 225, and 900, come to be the third number in this proportion; note that 15 *d.* is an hour in the Æquator, theſe multiplyed by 15, produceth 225 *German* Miles, and multiplyed by 60, produceth 900 *Engliſh* Miles, the Æquator paſſing ſo many Miles or Degrees in an hour, and ſo to run the whole circumference in 24 *hours*, 21600.

Thus much ſhall ſuffice to have ſpoken of the motion of the Earth, or the Sun, and of the Fixed Stars for the reſt.

Luna

Luna is the loweſt of all the Planets according to *Ptolomy*, or only a ſecondary Planet according to ſome *Copernicans*, finiſhes her courſe in 27 dayes, and almoſt 8 hours.

Mercury finiſhes his Revolution about the Sun in 87 dayes, 23 hours, 15 Minutes, 53 ſeconds according to *Kepler*, and is never diſtant from the Sun above 30 degrees.

Venus performes its courſe in 224 dayes, 16 hours, 49 minutes, 24 ſeconds, and is never diſtant from the Sun above 60 degrees.

Mars finiſhes his courſe in 2 years, or more ſtrictly in 686 days, 23 hours, 27 Minutes, and 30 ſeconds; or according to *Wing* in 1 year, 321 dayes, 23 hours, 32 minutes.

Jupiter moveth through the Zodiack in the ſpace of 11 years, 10 moneths, and almoſt 16 dayes, or according to *Kepler* in 4332 dayes, 12 hours, 20 Minutes, 25 ſeconds, according to *Wing* 11 years, 317 dayes, 14 hours, 49 minutes: His Satellites or Attendants circumvolveth him in time correſponding to their diſtances from him, the firſt and next him in 1 day, 18 hours; the third ſecond in 3 dayes, 13 hours; the in 7 dayes, 4 hours, the fourth and outermoſt in 16 dayes, 5 hours.

Saturn, the higheſt of all the Planets finiſheth his periodical courſe in 29 years, 5 moneths,

moneths, 15 dayes, according to *Alfraganus*, but more exactly according to *Kepler*, in 10759 dayes, 6 hours, 36 minutes, and 26 ſeconds, or according to *Wing*, *pag.* 63 *Harm.* in 29 years, 174 dayes, 4 hours, 58 minutes. Saturnes Moon, or Concomitant is obſerved to move about him in 16 dayes, and note that the circumrotation of the Sun upon his own Axis is in 26 dayes, and the Earth in 24 hours, (together with the reſt of the Planets that move about the Sun) Theſe all move from *Weſt* to *Eaſt*, alſo *Saturn* and *Jupiters* concomitants about them do keep the ſame courſe from Weſt to Eaſt.

And beſides theſe, it is not improbable but that every primary planet hath his proper Revolution upon his own Axis (as the Sun and Earth have theirs:) and that they have their Moons or Concomitants moving about them as our Earth hath: Nay, my Author adds that it is not unlikely but that the Sun being of the ſame matter with the Fixed Stars, they alſo may all have their Planets or Worlds moved about him, as our Sun hath his. And *Des Cartes* hath ſhown us (ſaith Doctor *Power*,) That every Fixed Star is a Sun, and is ſet in the Center of a *Vortex* or Planetary Syſteme as ours is; and that they are as far remote one off another, as ours is off them; and that all our whole Planetary *Vortex* ſhrinks into nothing

thing, if compared with these innumerable Systems above us.

No question but all these Planets are illuminated by the Sun (as is observed by the Telescope:) And that the Earth shines at a Distance with like splendor as her fellow Planets, as will easily appear by her illuminating the Darker part of the Moons subvolvane or lower Hemisphere, as is commonly seen a little before or after the change, for then to the Moon the appearance of the Earths light is near the full: But to make no further Digression. I come in the next place to shew the several magnitudes of these Cœlestial Bodies.

CHAP. V.

Concerning the Magnitude of the Planets and Fixed Stars, and their Distances from the Earth.

IT is not my Intention to shew you the Theory or way of Calculation, or finding these Magnitudes, and Distances, or the several motions of the Planets: These require a large Treatise of it self, and is learnedly and largely done by Mr. *Vincent Wing* in that excellent Book of his *Harmonicon Cœleste*,

Cœleste, to which I refer the Reader: And as I have done their Motions, so shall only *per Transitum* lay down their Magnitudes and Distances as I find them in Authors, and so proceed to questions of the Sphear, which is principally here intended. For their Magnitudes, take these Observations.

1. These Planets have their several Orbs wherein they move, the Semidiameter of which Orbs compared to the Semidiameter of the Earth have this proportion.

Of what Parts the Semidiameter of the Earth is one,			Deg.	Min.
of the same the Semidiameter of the Orb of	Luna	is	48	56
	Mercury		116	3
	Venus		641	45
	Sol		1165	23
	Mars		5032	4
	Jupiter		11611	31
	Saturn		17225	16

2. The Diameter of		Compared with the Diameter of the Earth, is as			
	Saturn		9	to	2
	Jupiter		32	to	7
	Mars		7	to	6
	Sol		11	to	2
	Venus		3	to	10
	Mercury		1	to	28
	Luna.		5	to	17

The Diameter of the Sun compared with the Diameter of the Moon beareth the same proportion that is betwixt 187 and 10,

The

The Diameter of a Star of the	1	Magnitude is to the Earths Diameter, as	119 to 4
	2		269 to 60
	3		25 to 6
	4		19 to 5
	5		19 to 36
	6		21 to 8

3. The Magnitude or Proportion of the Body or Globe of these Planets to the Globe or Body of the Earth according to the best observations of Mr. *Vincent Wing* in his *Harmonicon Cœleste*, *lib.* 3. *chap.* 25, 26, *fol.* 95, 96. are as follows.

Saturn, Jupiter, Sol	is greater than the Earth	$9\frac{2}{3}\frac{8}{1}\frac{3}{7}\frac{}{6}$	times.
		$9\frac{}{2}\frac{}{9}\frac{9}{8}\frac{3}{5}\frac{2}{9}$	times.
		343	times.

and greater than the Moon $15765\frac{1}{3}\frac{0}{0}\frac{3}{0}$ times.

Mars, Venus, Mercury, Luna	is lesser than the Earth	$55\frac{}{3}\frac{6}{2}\frac{6}{1}$	times.
		$26\frac{}{1}\frac{5}{2}\frac{0}{1}\frac{7}{6}\frac{7}{7}$	times.
		$117\frac{}{2}\frac{3}{7}\frac{7}{4}\frac{1}{4}$	times.
		$45\frac{2}{3}\frac{9}{0}$	times.

There are divers other Authors which do much differ from him in these Magnitudes, but I take him to be nearest the Truth.

The

The Magnitudes of the Fixed Stars, or their Proportions compared with the Globe of the Earth, which take as follows out of *Moxon* de Globis.

A Star of the 1 2 3 4 5 6 Magnitude is to the Globe of the Earth.

As 6859 to 64
As 19465109 to 216000
As 15625 to 216
As 6850 to 125
As 168515 9 to 46656
As 9261 to 512

Therefore bigger than the Earth.

Times.
107 1/2
90 1/2
72 1/2
54 1/2
36 1/2
18 1/2

This

This may seem incredible to visual sense, but if the vast Distance of those Huge Bodies, and the Diminutive quality of Distance be considered, reason will be rectified: For according to Mr. *John Dee*'s computation, the Starry Firmament is distant from the Earth 20081½ Semidiameters, which according to 60 Miles in a degree is 69006540 Miles: As is noted in its proper place.

And well may the Fixed Stars seem little to us; for Doctor *Power* saies, That if one stood in the Firmament to behold this Earth of ours, it could never be seen at all, and therefore if it were annihilated, would never be missed: So much doth Distance diminish the Object.

Now for their Distances from this Globe of the Earth, I shall give you them from *Varenius* in his own words, *Geog. gen. lib.* 1. *c.* 6.

Si quæras quantum Tellus & nos in Tellure existentes, distemus a Planetis, sciendum est, Non esse unam & eandem perpetuo distantiam sed singulis diebus mutari, & propterea Astronomi tres distantiarum gradus recensent minimam, mediam, maximam. Media Distantia Telluris a reliquis Planetis juxta plurimos Astronimos est hæc.

A Luna *distat Semidiametris suis*	60	
A Mercurio	110	
A Venere	700	
A Sole	1150	
A Marte	5000	*circiter.*
A Jove	11000	*circiter.*
A Saturno	18000	

Verum enim vero omnino incerta est Martis, Jovis, *&* Saturni *&* Stellarum *fixarum distantia, ob defectum parallaxis. In* Copernicana *Hypothesi distantia variatur non tantum a Planetarum motu sed etiam a motu, ipsius Telluris.*

John Seller's Atlas Cœlestis.

1. The *Sun* performs his Revolution upon his own Axis in 26 dayes.

2, *Mercury* performs his course in his Ellipsis in 88 dayes: His proper diurnal motion is 4 *d.* 5′. 12″. the Circuit of his Sphere is 12059773 miles, so that he wheels in a day 137040 miles in an hour, 5710 Miles, and in a Minute 91 miles.

3. *Venus* makes her periodical Revolution in her Ellipsis about the Body of the Sun in 224 dayes. It is from the Sun to the Sphere of *Venus* 3636104 miles, hence the Circuit of her Sphere is 22855911 miles. Her mean Diurnal Motion is 1 *d.* 36′. 8″. so that she moves in a day 101712 miles, in an hour 4238, in a minute 70.

4. The

4. The Earth accompliſhes her Revolution in 365 dayes, 5 hours, and 59 minutes, which according to *Copernicus* Syſteme is placed betwixt the Orbs of *Mars* and *Venus*. It is from the Sun to the Body of the Earth 5021896 Miles. The circuit of her *S*phere is 31560207 miles, her Diurnal motion 39 minutes, 8 ſeconds. A Degree of a great Circle upon the Earths Superficies is commonly reputed 60 miles, but by Mr. *Norwoods* experiment is found to be 69 miles.

5. *Mars* performs his Revolution about the *S*un in one year, 321 dayes, 22 hours, and 20 minutes. It is from the *Sun* to the Body of *Mars* 76 5292 miles. The Circuit of its *S*phere is 47993264 miles, his Diurnal motion 31 minutes, 27 ſeconds, ſo that he wheels in a day 69842 miles, in an hour 2910 miles, in a minute 48½ miles.

6. *Jupiter* runs his courſe in 11 *Egyptian* years, 315 days, 14 hours, and 30 minutes. It is from the *Sun* to *Jupiter* 26179152 Miles. The Circuit of his *S*phere is 164554670 miles, and his Diurnal motion about the *Sun* is 4 minutes, 59 ſeconds. Hence he wheeleth every day 17996 miles, every hour 1583 miles, and every minute 26 miles.

7. *Saturn* performeth one Revolution about the *Sun* in 29 *Egyptian* years, 162 dayes,

dayes, 1 hour, 58 minutes. It is from the *Sun* to *Saturn* 47833576 miles, the Circuit of his Sphere is 300668192 miles, his proper dayly motion is 2 minutes, 0 ſeconds. Therefore he wheeleth in a day 15959 miles, in an hour 1498, in a minute 25 miles. About his Body is a bright flat Ring which encompaſſeth him about.

The *Moon* is a ſecondary Planet, and retains the Earth for her Center, about which ſhe performs her Revolution in 27 dayes, 7 hours, 43 minutes. It is from the Center of the Earth to the Moon 203236 Miles. The Circumference of her Sphere is 1277483 miles. Her Diurnal Motion is 13 degrees, 10 minutes, 35 ſeconds, ſo ſhe wheeleth about in a day 46757 miles, in an hour 1948 miles, and in a minute 30 $\frac{1}{2}$ miles.

The Fixed Stars have a Motion or Circumrotation upon their own Axis, and beſides a Motion of Revolution from Weſt to Eaſt, which in a year is not leſs than 49 ſeconds, nor greater than 51 ſeconds, that it ſeems moſt probable that their Annual Motion is 50 ſeconds, 40 thirds, whence it follows that they compleat not one Degree in the Ecliptick ſooner than in 71 years, and $\frac{16}{1004}$ or 19 dayes, and 12 hours in a manner, but the whole Circle of 360 degrees they run not through in leſs than 25579

Sidereal

Sidereal years, which is *Annus Magnus Platonicus*, (though by the Antients computed to extend to 36000 years.)

But according to the Opinion and Observation of the Learned *Ticho Brahe*, as you may find in *Mathematical Recreations*, *fol.* 220. Their Distances are thus.

From the Center of the Earth	to the	Moon	is near	56 Semidiameters or 192116 miles
		Sun		1142 Semidiam. or 3924912 miles.
		Starry Firmament		14000 Semidiam. or 48184000 mil.

But according to Mr. *Dees* Computation 20081½ Semidiameters, or 69006540 Miles: From whence these Corollaries do follow.

1. Suppose that a man should go 20 Miles in Ascending towards the Heavens every day, he should be above 15 years before he could attain to the Orb of the Moon.

2. If a Millstone should descend from the place of the Sun 1000 Miles every hour, (which is above 15 Miles in a Minute, far beyond the proportion of Motion) it would be above 163 dayes before it would fall down to the Earth.

3. The Sun makes a greater way in one day than the Moon does in 20 days, becauſe that the Orb of the Suns Circumference is at the leaſt 20 times greater than the Orb of the Moon.

And thus much ſhall ſuffice to have ſpoken concerning the Number, the Names, the Order, the Motions, Magnitudes and Diſtances of theſe Heavenly Bodies, the *Planets*, and *Fixed Stars*.

CHAP. VI.

Of the Spheare.

I Come now to ſpeak of the *Spheare*, and of Queſtions of the *Spheare*, which I make the ſum and ſubſtance of *Aſtronomy*, as it ſtands in Analogy with *Geometry* in this place: For as *Surveying* is taking Heights and Diſtances at Land, *Navigation* at Sea, and are performed by the Reſolution of *Plain Triangles*: So this part of *Aſtronomy* is taking Heights and Diſtances in the Heavens (as is before hinted) and is performed by the Reſolution of *Spherical Triangles*, or by *Projection of the Sphear* it ſelf, which is the ground of theſe *Spherical Triangles*. And that I may proceed the more metho-

methodically, and be more clearly underſtood.

1. I ſhall firſt ſhew you what theſe Queſtions of the *Spheare* are, and how they may be termed taking Heights and Diſtances.

2. I ſhall ſhew you how theſe Queſtions may be reſolved.

I. By Trigonometrical Calculation, or the Reſolution of Spherical Triangles.

II. By Projection of the Sphear: 1. More Natural, By the Globe it ſelf. Or, 2. More Artificial, By Projection of the Spheare in *plano*, and ſo ſhew you theſe Queſtions may not only be reſolved by the Globe, but alſo by *Planiſpheres*, *Quadrants*, &c.

III. And laſtly, I ſhall inſiſt more largely concerning that principal and moſt uſeful queſtion of the Sphear, to wit, Dialling, or of finding the Hour of the Day by the Sun, and the Hour of the Night by the Moon and Stars, and ſhall herewith conclude the whole work.

SECT. 1.

Firſt then, The Aſtronomical *Queſtions of the* Sphear *are theſe,*

1. *For Heights.* To find

1. The Height or Elevation of the Pole, or Latitude of the place.

2. The Altitude or Height of the Sun or Stars above the Horizon at any time, which are called (particularly their Almicantaras, and their Meridian Altitude.)

2. *For Diſtances.*

3. The right Aſcenſion of the Sun or Stars may be alſo called an Height, becauſe it is the Meridian Altitude, or Height of the Equinoctial above the Horizon: But may be alſo called a Diſtance, becauſe the Diſtance or Number of Degrees contained betwixt the firſt Degree of *Aries*, and the Degree of the Equinoctial that comes to the Meridian with the Sun or Stars is the Right Aſcenſion of the Sun or Stars in Degrees and Minutes.

4. The Oblique Aſcenſion of the *Sun* or *Stars*, though it ſounds like an Height, is more properly a Diſtance, becauſe it is the Diſtance in Degrees of the Equinoctial betwixt the firſt point of *Aries*, and that very point of the Equinoctial that riſes or cuts the Horizon with the *Sun* or *Stars*.

5. The Aſcenſional Difference is in plain *Engliſh* no more but the Diſtance betwixt the Oblique and Right Aſcenſion.

6. The *Suns* Meridian Diſtance North or *South* from the Equinoctial, or of a ſtar from the Ecliptick is called their Declination.

7. The

7. The Suns diſtance from any of the 12 ſigns is called his place in the Zodiack.

8. His Diſtance of riſing or ſetting from the due point of Eaſt or Weſt, is called his Amplitude.

9. The *Suns* Diſtance at any time from any point of the Horizon is called his *Azimuth*, or Angle of the *Suns* poſition.

10. The *Suns* diſtance from the next Equinoctial point *Aries* or *Libra*, requires no Explanation.

Laſtly, His Diſtance from the Meridian ſhews the hour of the day, and the Moon and ſtars diſtance from the Meridian ſhews the hour of the night. And all theſe in theſe reſpects may be properly called Heights and Diſtances in *Aſtronomy*.

SECT. 2.

Therefore in the ſecond place I ſhall ſhew you how to find theſe Heights and Diſtances.

1. By Trigonometrical Calculation, or the Reſolution of Sphærical Triangles.

2. By Projection of the Sphear, either Globular, or in *plano*, in whole, as in the Globe it ſelf and Planiſphears; or in part, as in Semicircles, Quadrants, and the like.

For the firſt of theſe, to ſhew you how theſe Queſtions may be reſolved by the Reſolution of Spherical Triangles, I ſhall endeavour to do theſe two things.

1. I ſhall ſpeak ſomething concerning Spherical Triangles, and the way how to make and meaſure them.

2. I ſhall give you the Trigonometrical proportions how to find theſe ſeveral Aſtronomical Heights and Diſtances before-mentioned by Sines and Tangents.

SECT. 3.

Of Spherical Triangles.

As *Plain Triangles*, ſo *ſpherical* conſiſt of ſix parts, *viz.* three Angles, and three ſides; the Angles of both are meaſured and protracted by a line of Chords. The difference betwixt them lies only in this, that the ſides of Plain Triangles are Right lines drawn by the fiducial edge of a Ruler, and meaſured by a Line of equal parts: whereas the ſides of ſpherical Triangles are Arches of great Circles (that is Arches of any Circle to the ſame Radius) protracted upon the Center of the ſame Arches or Circles, and meaſured by a Line of Chords after the ſame manner, that all Circles and Arches are meaſured by Degrees and Minutes.

And

And as the Proportions betwixt the ſides and Angles of Plain Triangles are found by numbers and ſines: As for Example, The Artificial ſine of any Angle is proportionable to the Logarithmetical Number of its oppoſite ſide; and on the contrary, the Logarithmetical Number of any ſide is proportional to the Artificial ſine of its oppoſite Angle: So are the ſides of ſpherical Triangles (being Arches of great Circles; and the Angles alſo meaſured at their Quadrantal Diſtance, or at the Diſtance of the Radius from the Angular point,) and the proportion betwixt theſe ſides and Angles are found by ſines only by this Rule. *Harm. Cœl. fol.* 10.

In every, or in any ſpherical Triangle the ſines of the ſides are proportional to the ſines of their oppoſite Angles, and the ſines of the Angles are proportional to the ſines of their oppoſite ſides.

And in theſe particulars lies the difference betwixt plain and ſpherical Triangles, and betwixt the protracting or making, and the meaſuring of the ſame: More particularly,

1. *To meaſure or take the Quantity of any angle of a ſpherical Triangle, and to protract the ſame by the Projection.*

Lay a Ruler to the Angular point, and

the Extremity of the ſides containing the Angle, they being continued to Quadrants, and note where the Ruler cuts the Meridian or outward Circle, at both which places make marks upon the Meridian: The Diſtance between thoſe two marks meaſured upon your Line of Chords, ſhall give you the quantity of the Angle required.

And to Protract an Angle, do thus.

Firſt, Taking 90 degrees or Radius from your Line of Chords, deſcribe a Circle or part of a Circle, ſo much of it as you have occaſion for, and making the Center to be the Angular point, prick out in the circumference ſo many Degrees and Minutes from the ſame Line of Chords as you would have your Angle to contain: Then at the diſtance of the Radius from theſe marks in the ſame Circumference placing one foot of your Compaſſes, with the other ſtrike an Arch through the mark and the Center, do the like with the other mark: Theſe Arches at the Center ſhall include an Angle of a ſphærical Triangle, containing the ſame Number of Degrees as were markt out in the Circumference. The like Ratio holds in any other Angle. But,

2. *To*

2. *To Measure, or take the quantity of any side of a Spherical Triangle,*

And to protract the same, is more difficult, and requires in the first place (seeing that all Circles have their proper Poles) That the Poles of those Circles (whereof those sides are Arches) be found out.

3. *Now to find out these Poles.*

Note that the Pole of every great Circle is 90 degrees, or a Quadrant of a Circle and distant from the Circle it self upon that line which cutteth the Circle at Right Angles. Thus in a Projection of the Sphear the Poles of all the hour-Circles are upon the Æquinoctial, and the Poles of all the Azimuths upon the Horizon: The Poles of the Æquinoctial are the Poles of the World; and the Poles of the Horizon, the Zenith and Nadir. Now to find the Pole of any Arch continued to a Quadrant, lay a Ruler by the extream ends of the said Arch, and mark where it cuts the Meridian or outward Circle: Then take 90 Degrees from your line of Chords, and set off that distance from the said mark; and lastly, lay a Ruler from the former end of the Arch to this latter mark, and where this crosses the Circle

which cuts the Arch at right Angles, there is the Pole of the ſaid Arch. To make it more plain by an Example.

Suppoſe you would find the Pole of the Hour-circle PDS in the Projection, lay a Ruler upon P and D, it will cut the Meridian or outward Circle in *e*. Then take 90 degrees of your Line of Chords, and ſet them from *e* to *f*. A Ruler layd from P to F will cut the Æquinoctial in *y*, ſo is *y* the Pole of the Circle PDS, and ſo of any other as the Pole of ZGN is ☉, *&c.*

4. *Thus having found the Pole of a Circle, or of the ſide of a Spherical Triangle, which is the Arch of a Circle, To meaſure the ſaid ſide.*

Lay a Ruler upon the ſaid Pole, and to the extream ends of the ſide of a Triangle, note where the Ruler ſo laid cuts the Meridian or outward Circle at both ends of the ſide: That Diſtance taken in your Compaſſes, and meaſured upon the Line of Chords, will give you the Quantity of the ſide of the Triangle. As for Example in the Projection: Let it be required to find the ſide EZ of the Triangle ZEP, lay a Ruler to ☉, (the Pole of the Circle ZEN,) and the Angular point E, it will cut the Meridian in M, and a Ruler laid to Z will cut

the

the Meridian in Z, so the distance MZ taken in the Compasses, and measured upon the Line of Chords, will be found to contain 78 degrees, and such is the quantity of the side ZE.

5. *Now to Protract a side of a Spherical Triangle, is taught by measuring of it.*

For first you are to find the Pole, then laying your Ruler from the Pole by the extremity of one end of the side, make a mark upon the Meridian or outward Circle, and make another mark at the extremity of the other end of the side; and lastly find out the Center of the Circle to the same Radius, which will cut both the extremities of the side required, and it is done.

SECT. 4.

Having thus shewn you how to protract and measure either Angle or side of a Spherical Triangle, and withal given you *pag.* 343 the Rule to find out the proportions; or by some of the 6 parts given, to find the rest, *viz.* That the sines of their sides are proportional to the sines of their opposite Angles & *e contra*, which Rule is so general, that thereby you may resolve all, or most questions of the Sphere or Spherical

Tri-

Triangles: yet I think it not amiſs to lay down the particular proportions by which theſe ſeveral Aſtronomical Queſtions of the Sphear are reſolved and found by the lines of Artificial Sines and Tangents, or Logarithmetical Arithmetick.

How to find the Suns place
And his Declination,
His Meridian Altitude, and thereby,
The Latitude.

All theſe you will find Taught in *Navigation*, pag. 254, 257, and 258, whereunto I refer the Reader: They being there handled as they are *Data*, or things given: And therefore ſome of theſe being given, I ſhall ſhew you here how they and other poſtulata's or quæſita's may be found in theſe following queſtions.

PROBL. I.

By the Suns ſhadow to find his Altitude.

Take a two-foot Rule, or any other ſtaff divided into equal parts: And as the length of the ſhadow is to the length of the Rule or Staff (held perpendicular to the Horizon) upon the line of Numbers; ſo is the Tangent of 45, or Radius to the Suns Height required: As ſuppoſe the ſtaff divided into 100 parts, the ſhadow is 83 of theſe

these parts, the extent betwixt 100 and 83 in the Line of Numbers will reach from Radius, or 45 upon the Line of Tangents to 50 *d.* 18 *m.* 26 seconds, the Suns Height above the Horizon.

PROBL. 2.

To find the Suns Declination. vide pag. 258.

PROBL. 3.

To find the Suns place you have also 257, But shall add this other Analogy, *The Suns greatest Declination, and his present Declination given, to find his Distance from the next Æquinoctial point, and thereby his place in the Ecliptick.*

As his greatest Declination (which is alwaies 23 *d.* 30 *m.*) is to Radius or sine 90, so is his present Declination (which is also to be given) to his Distance from the next Æquinoctial point: Suppose the Declination 11 *d.* 30 *m.* his distance is 30 *d.* from the next Æquinoctial point, which is *Aries*, therefore his true place is just leaving *Aries*, and entring *Taurus*.

PROBL.

PROBL. 4.

The Suns greateſt Declination, and Diſtance from the next Æquinoctial point given, To find his Right Aſcenſion.

As the Radius is to the Tangent of his Diſtance, *&c.* ſo is the Co-ſine of his greateſt Declination to the Tangent of his Right Aſcenſion. Example, As 90 *d.* is to 66 *d.* 30, ſo is Tangent of 30 *d.* to the Tangent of 27 *d.* 50 minutes.

PROBL. 5.

The Suns greateſt Declination, and his preſent Declination given, to find his Right Aſcenſion.

As the Tangent of his greateſt Declination (23 *d.* 30 *m*) is to Radius, ſo is the Tangent of his preſent Declination (which ſuppoſe 11 *d.* 30 *m.*) to the Sine of his Right Aſcenſion 27 *d.* 50 *m.*

PROBL. 6.

To find the Oblique Aſcenſion.

If the Declination be North, ſubſtract the Aſcenſional Difference from the right Aſcenſion, and it will give the Oblique Aſcenſion. If the Declination be South, add

add the ſame Aſcenſional Difference and Right Aſcenſion together, and the Sum will be the Oblique Aſcenſion.

PROBL. 7.

To find the Aſcenſional Difference, the Latitude and Declination given.

As the Co-Tangent of the Latitude, (which ſuppoſe 38 *d.* 30 *m.* the Latitude being 51 *d.* 30 *m.*) is to the Tangent of the Suns Declination (20 *degrees*) ſo is the Radius (90 *d.*) to the ſine of the Aſcenſional Difference (27 *d.* 14 *m.*) and ſo much the Sun riſeth or ſetteth before or after ſix, according as the Declination is North or South, and this 27 *d.* 14 *m.* converted into time (by allowing 15 degrees to an hour, and one degree for 4 minutes of time) is 1 *h.* 49 *m.* and ſo much doth the Sun riſe or ſet before or after the hour of ſix, according to the time or ſeaſon of the year.

PROBL. 8.

The Latitude of the Place, and Declination of the Sun given, To find his Amplitude.

As the Co-ſine of the Latitude (38 *d.* 30) is to Radius (90) ſo is the Sine of the Declination

clination (ſuppoſe 20 *d.*) to the ſine of the Amplitude (33 degrees 20 minutes.)

PROBL. 9.

To find the Suns Altitude, and firſt his Meridian Altitude, his Declination (only) given.

If the Declination be North, add the ſame to the Complement of the Latitude, and the Total of both ſhall be the Meridian Altitude. If the Declination be South, ſubſtract the ſame from the Complement of the Latitude, and what remains ſhall be the Meridian Altitude.

PROBL. 10.

The Latitude and Declination given, to find the Suns Altitude at 6 a Clock.

As the Radius (90 *d.*) is to the ſine of the Suns Declination (20 *d.*) ſo is the ſine of the Latitude (51 *d.* 30 *m.*) to the ſine of the Suns Altitude at 6, *viz.* 15 *d.* 30 *m.*

PROBL. 11.

The Latitude and Declination given, To find the Suns Altitude when he is upon the true Eaſt and Weſt points.

As the ſine of the Latitude (51 *d.* 30 *m.*) is to the ſine of the Declination (20 *d.*) ſo is the Radius (90 *d.*) to the ſine of the Suns

Suns Altitude, being due East or West (23 *d.* 55 *m.*)

PROBL. 12.

To find the Suns Altitude at any time assigned.

First, Say as the Radius is to the Cotangent of the Elevation, so is the sine of the Suns distance from six to the Tangent of an Ark, which being substracted out of the Suns distance from the Pole. I say again,

Secondly, As the Co-sine of the Ark found is to the Co-sine of the Residue of the Suns Distance from the Pole, so is the sine of the Elevation to the sine of the Height required.

PROBL. 13.

To find the Angle of the Suns Position, having given the hour from Noon: The Latitude, and the Suns Altitude.

As the Co-sine of the Suns Altitude, (which suppose 78 *d.*) is to the sine of the hour from Noon (35 *d.* 36 *m.*) so is the Co-sine of the Latitude (38 *d.* 30 *m.*) to the sine of the Angle of the Suns Position at the time of the Question, (21 *d.* 45 *m.*)

The

PROBL. 14.

The Latitude of the place, and Declination of the Sun being given, To find the Angle of the Suns poſition at his Riſing.

As the Co-ſine of the Declination (70 *d.*) is to Radius (90 *d.*) ſo is the ſine of the Latitude (51 *d.* 30 *m*) to the ſine of the Angle of the Suns Poſition at the time of his riſing.

PROBL. 15.

Having the Suns Altitude, His Declination, and Diſtance from the Meridian (or the hour of the day) given, to find his Azimuth.

As the Co-ſine of the Suns Altitude, (which ſuppoſe 64 *d.* 4 *m.*) is to his diſtance from the Meridian (61 *d.*) or 4 hours, 4 *m.* before noon (*viz.* 7 hours 56 *m.*) ſo is the Co-ſine of the Declination (78 *d.* 30 *m.*) to the Suns Azimuth from the South Eaſtwards, (*viz.* 72 *d.* 22 *m.*)

PROBL. 16.

The Latitude of the place and Declination of the Sun given, To find what Azimuth the Sun ſhall have at ſix a Clock.

As the Co-ſine of the Latitude (38 *d.* 30 *m.*) is to Radius (90 *d.*) ſo is the Co-tangent

Tangent of the Declination (70 *d.*) to the Tangent of the Suns Azimuth counted from the North part of the Meridian (77 *d.* 14 *m.*)

PROBL. 17.

The Declination, Altitude, and Azimuth of the Sun being given, To find the Hour of the Day.

As the Co-ſine of the Declination (78 *d.* 30 *m.*) is to the Co-ſine of the Altitude (64 *d.* 4 *m.*) ſo is the Azimuth (72 *d.* 22 *m.*) to 61 *d.*) the hour of the day. This 61 *d.* converted into time gives 4 hours, 4 minutes, which taken from 12 hours, the Remainder is 7 hours, 56 minutes before Noon: The time of the day required.

Mr, Leyburns *Analogy is this,*

As the Co-ſine of the Declination (70 *d.*) is to the ſine of the Azimuth (146 or 34 degrees) ſo is the Co-ſine of the Altitude (78 *d.*) to the ſine of the hour from Noon 35 *d.* 36 *m.*

CHAP.

CHAP. VII.

THus you have the ſeveral Analogies and Proportions, whereby may be reſolved theſe moſt uſeful Queſtions or Problems of the Sphear by the *Rule of Three*, wrought either by *Logarithmes*, *pag.* 31. or by the Lines of Artificial Numbers, Sines, and Tangents, *pag.* 41. taking this Obſervation along with you.

That (*As*) denotes, or is put before the firſt Number: (*Is to*) the ſecond: (*So is*) denotes the third: and (*To*) denotes or precedes the fourth, ſo that multiplying the ſecond by the third, and dividing the Product by the firſt, produceth the fourth in the Direct *Rule of Three*, *pag.* 19. and multiplying the Firſt by the Second, and dividing by the Third, produceth the Fourth by the Indirect or Reverſe *Rule of Three*, *pag.* 19. where you may find both the ſaid Rules.

Further you may obſerve, That the Suns Declination and Altitude is Aſſiſtant to find the Latitude: His Amplitude and Azimuth, to find the variation of the Compaſs: The Aſcenſional Difference to find the time of the Suns Riſing and Setting: The right Aſcenſion

Aſcenſion of the Sun and Stars is Aſſiſtant to find their coming upon the Meridian, and the hour of the Night. The Latitude, Declination, and the Suns Altitude, and Azimuth is Aſſiſtant and ſubſervient to the finding the hour of the day: Theſe two laſt put together, that is, the finding of the hour of the Day and Night is that which we call Dialling, concerning which, you will find ſomething particularly in the Concluſion of this ſmall Treatiſe.

Having now ſhown you how thoſe queſtions of the Sphear may be reſolved by *Trigonometrical Calculation*, and their ſeveral Analogies and Proportions alſo, by which they may be found by Arithmetical Operation. I come now to ſhew you how theſe may be done by ſeveral Projections.

Firſt, By the Globe it ſelf, being the moſt natural projection of the Sphear of all others whatſoever.

Secondly, By *Theakers Planiſphear*, an Inſtrument of good uſe for this purpoſe, called a light to the Longitude.

Thirdly, By Mr. *Foſters* Quadrant, that is, a Quadrant with hour Lines.

Fourthly, By the Projection of the Sphear in plano. And,

Laſtly, I ſhall particularly inſiſt more at large upon that Queſtion concerning finding of the hour of the day by the Sun, and

of the hour of the Night by the Moon and Stars, which may be properly termed Dialling, and with that conclude what I here intend in this Manual.

SECT. 1.

The Uſe of the Globe in queſtions of the Sphear.

Thoſe who have a mind to be throughly inſtructed in the general Uſe of the Globes, both Cœleſtial and Terreſtrial, I recommend to them that moſt compleat Book written by Mr. *Joſeph Moxon* concerning the Uſe of the Globes in the ſolution of Aſtronomical, Geographical, Nautical, Aſtrological, Gnomonical Problems, and in the Reſolution of Spherical Triangles at large: What I here intend is after a very brief manner, to ſhew you how theſe Queſtions of the Sphear may be reſolved by the Globe eaſily and ſpeedily.

Firſt then, You are to find the Latitude of the place where the queſtion is propounded.

Which to do by Obſervation is taught *pag.* 255. but if you deſire to do it by the Globe, you are to bring the particular place to the Brazen Meridian, and the degrees reckoned thereupon from the Equator to the

the place whoſe Latitude you ſeek, is the Latitude whether North or South: And for the Longitude likewiſe you may find it by Obſervation, as in *pag.* 260. and by the Globe. Obſerve what Degrees of the Equator comes to the Meridian with the place, the ſame is the degrees of Longitude from the firſt Meridian.

In the next place you are to rectifie your Globe fit for uſe: Which to do,

1. When you rectifie the Globe to any particular Latitude, you muſt move the Brazen Meridian through the Notches of the Horizon, till the ſame Number of Degrees accounted on the Meridian from the Pole (towards the North point in the Horizon, if in North Latitude, and towards the South point, if in South Latitude) come juſt to the edge of the Horizon.

2. Next rectifie the Quadrant of Altitude, and ſcrew the edge of the Nut that is even with the graduated edge of the thin plate to the ſame Number of Degrees as the Latitude is accounted on the Meridian from the Equinoctial, which will be the Zenith of the place.

3. Bring the degree of the Ecliptick the Sun is in that day to the Meridian, (which you ſhall learn to know hereafter) and then

then turn the Index of the Hour-Circle to the hour of 12 on the South-ſide the Hour-Circle, if in North Latitude & *contra*; and then is your Hour-circle alſo rectified, fit to uſe for that day.

Laſtly, To rectifie the Globe to correſpond in all reſpects with the poſition of the Sphear, you muſt with a Needle placed in the bottom ſet the 4 quarters of the Horizon *Eaſt*, *Weſt*, *North*, and *South*, agreeable to the 4 Quarters of the World, and the Horizon by a Level you may ſet parallel to the Horizon of the World.

And poſiting the degree of the Ecliptick the Sun is in to the Height above, or depth below the Horizon the Sun hath in Heaven: Your Globe is made correſpondent in all points with the frame of the Sphear, for that particular Time and Latitude.

The Globe thus rectified, you may find all the foremention'd Problems on this manner.

1. Find the day of the Moneth in the wooden Horizon, and over againſt it you will find the degree of the Ecliptick, which the Sun is in that day: As ſuppoſe at *London* (where the Pole is elevated 51 ½ degrees of Northern Latitude, the Globe rectified for the ſame Latitude, as alſo the Hour-Circle, and Quadrant of Altitude) over againſt

againſt the 10*th* of *May* you will ſee the Suns place in the Zodiack is 29 degrees of ♉ *Taurus*.

2. Bring this degree which is the Suns place to the Eaſt-ſide of the Horizon, and its diſtance from the Equinoctial, or from the due Eaſt point is the Suns Amplitude, or ſo much as the Sun riſeth before or after the hour of ſix 33 *d.* 20 *m.* from the Eaſt-point Northwards in this Example.

3. Look upon the Hour-Circle, and the Index will point to the hour of Sun-riſing, which in this Example is 12 Minutes after 4 in the morning.

4. Obſerve what degree of the Æquinoctial riſeth with the Suns place, and that is his Oblique Aſcenſion, reckoning from the Vernal colure in the Cœleſtial, and from the firſt Meridian in the Terreſtrial Globe.

5. As you move the Globe round upon his Axis Weſtward, you may (at any time or hour which the Index of the Hour-Circle points to) know the Suns Altitude at that time, by bringing the Quadrant of Altitude to cut the Suns place: As at 53 *m.* paſt 8 in the morning, you will find it juſt 40 degrees high above the Horizon: And,

6. The Quadrant of Altitude at the Horizon will ſhew his Angle of Poſition, or upon what point of the Compaſs it is.

7. The Suns Azimuth is the ſame with his Angle of Poſition.

8. And his Almicanter the ſame with his Altitude.

9. Bring the Suns place to the Meridian, and there the degrees intercepted betwixt the Suns place and the Horizon, reckoning upon the Brazen Meridian, is the Suns Meridian Altitude: In this Example, (50 *d.* 30 *m.*)

10. The Diſtance betwixt the Suns place in the Ecliptick, and the Equinoctial is the Suns Declination reckoned alſo upon the Meridian: In this Example (20 *d.* 5 *m.*)

11. The Degree of the Equinoctial which comes to the Meridian with the Suns place, is his right Aſcenſion, reckoning from the Vernal Colure, or firſt Meridian.

12. The number of degrees comprehended betwixt the Oblique and Right Aſcenſion, is the Aſcenſional Difference reckoned upon the Equinoctial.

13. At any Degree of Altitude, either before Noon, or after Noon, the Index of the hour-Circle points to the hour of the day.

14. And laſtly, when the Suns place comes to the Weſt ſide of the Horizon, the Index of the Hour-Circle will point to the hour of Sun-ſetting, and alſo will ſhew his Oblique Deſcenſion, and upon what

Azimuth

Azimuth or point of the Compaſs it ſets, and how much before or after ſix.

What is here ſaid of the Sun, the ſame is to be underſtood of any Star, uſing the ſaid Star as you do the Suns place: As to Inſtance in theſe particulars: The Globe rectified as before.

1. By the Meridian Altitude of any Star, to find the Elevation of the Pole: Obſerve by a Quadrant or other Inſtrument the Meridian Altitude, and bringing the Star to the Meridian, place it upon the Globe to the Altitude obſerved, ſo ſhall the Number of Degrees intercepted between the Pole and the Horizon, be the elevation of the Pole.

2. To find the Right Aſcenſion of any Star, bring the Star to the Meridian, and the Number of degrees betwixt the Vernal colure, and the Meridian its Right Aſcenſion; in like manner for his Oblique Aſcenſion, or oblique Deſcenſion.

3. To find the Declination of a Star, bring the Star to the Meridian and count the Number of Degrees betwixt the ſaid Star, and the Equinoctial, the ſame is the Stars Declination be it North or South.

4. The Azimuth and Almicantara, or which is all one, The Altitude and Angle of Poſition of any Star is found the ſame way as the Suns Azimuth and Almicantara, as in 5, 6, 7, 8, *fol.* 369.

5. To know any Star in Heaven, Firſt, having a Theodolite, with a moveable Index, upon the Index fix a two-foot Rule, or ſuch a thing perpendicular; and at the top of the Rule hang a Quadrant, then placing your Theodolite true North and South by help of a Needle, or a Meridian Line, you may by the Quadrant take the Almicantara or Altitude; and the Index of the Theodolite will ſhew the Azimuth or Angle of Poſition: And obſerve by the Globe what Star there hath the ſame Azimuth and Almicantara at the ſame time, and that is the ſame Star which you obſerved in Heaven: For if every Star on the Globe had a hole in the midſt, and your Eye were placed in the Center of the Globe, you might by keeping your Eye in the Center, and looking through any Star on the Globe, ſee the ſame Star in the Heaven, which that on the Globe doth repreſent; for from the Center of the Globe there proceeds a ſtrait Line through the Star on the Globe even to the ſame Star in Heaven.

6. The Meridian Altitude of a Star is found upon the Globe by bringing the ſaid Star to the brazen Meridian, and reckoning how many degrees it is above the Horizon, the Globe rectified as before ſhown.

7. And by the Hour-Circle, turning about the Globe, you may obſerve at what hour

hour any Star will be upon the Meridian, to wit, when the ſame Star on the Globe touches the Brazen Meridian.

8. The Globe rectified, and the Altitude of a Star found by a Quadrant in the Heavens, and the ſame Star brought to the ſame degree of Altitude upon the Globe, by the Quadrant of Altitude, the Index of the hour-Circle will point to the hour of the night.

9. The Riſing and Setting of the Stars are found as the riſing and ſetting of the Sun: But this riſing and ſetting admits of a threefold diſtinction:

1. Coſmical, when any ſtar riſeth with the Sun, it is ſaid to riſe Coſmically, and when any ſtar ſets when the Sun riſeth, it is ſaid to ſet Coſmically.

2. *Acronical*, The ſtars that riſe when the Sun ſets are ſaid to riſe Acronically, and the ſtars that ſet with the Sun are ſaid to ſet Acronically.

3. *Heliacal*, when a ſtar formerly in the Suns Beams gets out of the Sun Beams, it is ſaid to riſe Heliacally, and when a ſtar formerly out of the Suns Beams gets into the Suns Beams, it is ſaid to ſet Heliacally.

And what ſtars do riſe or ſet any of theſe waies, and when may be eaſily found by the Globe, as in *Moxons lib.* 2. *Probl.* 35, 36, 37, which I ſhall not here inſiſt upon.

Laſtly, I ſhall ſhew you how to find the Longitude and Latitude of a ſtar, and ſo conclude as to the Cœleſtial Globe.

If the ſtar you enquire after be on the North-ſide the Ecliptick, you muſt elevate the North Pole $66\frac{1}{2}$ degrees: If on the South-ſide you muſt elevate the South Pole as much, then bring the ſolſtitial colure to the Meridian on the North ſide the Horizon, and ſcrew the Quadrant of Altitude to the Zenith, which will be $23\frac{1}{2}$ degrees from the Pole of the World, ſo ſhall the Ecliptick lye in the Horizon, and the Pole of the Ecliptick alſo lye under the Center of the Quadrant of Altitude: Now to find the Longitude of any ſtar, turn the Quadrant of Altitude about till the graduated edge of it lye on the ſtar, and the degree in the Ecliptick that the Quadrant touches is the Longitude of that ſtar: And the degree of the Quadrant of Altitude that touches the ſtar is the Latitude of the ſtar, ſo the Longitude of *Marchab* a bright ſtar in the Wing of *Pegaſus* will be found ♐ 18 *d.* 56. and its Latitude 19 *d.* 26 *m.*

What hath hitherto been ſaid, concerns the Cœleſtial Globe, I ſhall add a few propoſitions proper to the Terreſtrial, which together will contain the principal Problems to be performed by the Globe.

To

1. *To find the Distance between any two places on the Terrestrial Globe.*

The graduated edge of the Quadrant of Altitude applyed to both the places, will shew the number of Degrees betwixt them; or with a pair of Compasses pitch one foot in one of the places, and the other foot in the other place, and keeping your Compasses at that distance, apply the feet to the Equinoctial Line, and you will find the number of Degrees comprehended between them: As suppose the places were *London* and *Jamaica*, you will find 68½ degrees, which multiplyed by 60, makes 4110. *English* Miles.

2. *To find upon what point of the Compass any two places are scituate one from another.*

Find the two places on the Terrestrial Globe, and see what Rumb passes through them, or if no Rumb pass immediately through both, take that Rumb which runs most parallel to both the places.

3. *To know what a Clock it is in any other part of the Earth.*

Note that those places to the Eastward

of us have their day begin ſooner than ours, and thoſe to the Weſt-ward later; therefore firſt bring the place of your own habitation to the Meridian, and the index of the Hour-Circle to 12, then bring the other place to the Meridian, and the Index will point to the hour in the other place, be it ſooner or later.

4. *Any place on the Terreſtrial Globe given, to find its Antipodes.*

Bring the given place to the Meridian, ſo you may find its Longitude and Latitude, then turn about the Globe till 180 degrees of the Æquator paſs through the Meridian, and keeping the Globe to this poſition, number on the Meridian 180 degrees from the Latitude of the given place, and the point juſt under that degree is the Antipodes; or bring the given place to the North or South point of the Horizon, and the point of the Globe denoted by the oppoſite point of the Horizon is the Antipodes of the given place.

5. *To find the Perecy of any given place.*

You muſt underſtand the Perecy of any place are ſuch as dwell about the ſame Hemiſphere in the ſame Parallel from the Æquator, the one Eaſtern, the other Weſtern, directly oppoſite to one another: There-

Therefore bring your place to that ſide the Meridian which is in the South notch of the Horizon, and follow the parallel of that place on the Globe till you come to that ſide the Meridian which is in the Northern notch of the Horizon, and that is the Perecy of your place.

7. *To find the Antecy of any given place.*

The Antecy are ſuch as dwell one againſt another, having the ſelf ſame Meridian, and equal diſtance from the Æquator, the one in the Northern, the other in the Southern Hemiſphere, therefore bring your place to the Meridian, and find its Latitude: Then if it have North Latitude, count the ſame number of Degrees on the Meridian from the Æquator South-wards: but if it have South Latitude, count the ſame number of Degrees from the Æquator Northwards, and the Point of the Globe directly under that number of Degrees is the Antecy of your place.

8. *How to hang the Terreſtrial Globe in ſuch a Poſition, that by the Suns ſhining upon it, you may with great delight at once behold the Demonſtration of many Principles in Aſtronomy and Geography.*

Take the Terreſtrial Ball out of the Horizon, and faſten a Thread on the Brazen

Meridian to the Degree of the Latitude of your place: By this thread hang the Globe where the *Sun*-beams may have a free acceſs to it: Then direct the Poles of the Globe to their proper Poles in Heaven, and with a thread faſtned to either Pole, brace the Globe ſo that it do not turn from his poſition: Then bring your habitation to the Meridian, ſo ſhall your Terreſtrial Globe be rectified to correſpond in all reſpects with the Earth it ſelf, ſo that with great delight you may behold

1. How the counterfeit Earth (like the true one) will have one Hemiſphear *Sun*-ſhine-light, and the other ſhadowed by the ſhining Hemiſphere, you may ſee that it is day in all places that are ſcituate under it, for on them the *Sun* doth ſhine: And that it is Night at the ſame time in thoſe places that are ſcituated in the ſhadowed Hemiſphear; for on them the *Sun* doth not ſhine.

2. If in the middle of the enlightned Hemiſphear you ſet a *Spherick* Gnomon perpendicularly, it will project no ſhadow, but ſhew that the *Sun* is juſt in the Zenith of that place, that is directly over the heads of the Inhabitants.

3. If you draw a Meridian Line from one Pole to the other juſt through the middle of the enlightned Hemiſphear, in all places under that Line, it is Noon; In thoſe places

ces ſcituate to the Weſt it is Morning, for with them the Sun is Eaſt; and in thoſe places ſcituate to the Eaſt, it is Evening, for with them the Sun is Weſt.

4. So many degrees as the Sun reaches beyond either the North or South Pole, ſo many Degrees is the Declination of the Sun either Northwards or South-wards, and in all thoſe places comprehended in a Circle deſcribed at the termination of the Sun-ſhine about that Pole, it is alwaies day till the Sun decreaſe in Declination; for the Sun goes not below their Horizon, as you may ſee by turning the Globe about upon its Axis: And in the oppoſite Pole at the ſame diſtance the Sun-ſhine not reaching thither, it will be alwaies Night till the Sun decreaſe in Declination, becauſe the Sun riſeth not above their Horizon.

5. If you let the Globe hang ſteddy, you may ſee on the Eaſt-ſide of the Globe in what places it grows Night; and on the Weſt-ſide of the Globe how by little and little the Sun encroaches upon it, and therefore there makes it day.

6. If you make of paper or parchment a narrow Circle to begirt the Globe juſt in the Equinoctial, and divide it into 24 equal parts to repreſent the 24 hours of day and night, and mark it in order from 1, 2, 3, to 12, and then begin again with 1, 2, 3, to

to other 12, you may by placing one of the Twelves upon the Equinoctial under the Meridian of your place have a continual Sun-dial of it, and the hour of the day given on it at once in two places, to wit, where the enlightned Hemiſphere is parted from the ſhadowed both on the Eaſt and Weſt-ſide of the Globe; much more might be ſaid on this ſubject, but the ingenious Artiſt may of himſelf find out diverſities of other ſpeculations.

And thus much ſhall ſuffice to have ſpoken concerning the uſe of the Cœleſtial and Terreſtrial Globes, and ſo I ſhall return from this Digreſſion to my former propoſed Method; and ſhew you how moſt of thoſe formerly mentioned queſtions of the Sphear, or Spherical Problems, *pag.* 398 may be reſolved.

SECT. 2.

By Theakers Planiſphear.

This Inſtrument conſiſts of two Sphears or Plains, the lowermoſt and fixed, wherein is deſcribed three ſeveral hour-Circles, the uppermoſt divided into 24 hours, wherein is reckon'd the Right Aſcenſion of the Sun or Stars in hours and minutes: The next is called *Aries* hour-Circle, having ♈ ſtanding betwixt 24 of the outward hour-Circle, and 12 in the inward or third hour-

hour-Circle; in this is reckon'd the time of the Sun and stars rising and setting, or of the stars coming to the South: The third hour-Circle is divided into twice twelve, and in this is reckoned the difference of time, and next to this is the Æquinoctial Circle divided into 360 degrees: In the upper moving Sphear or Plain is drawn, First, The Circle of Moneths, placing ♈ *Aries* at the 10*th.* day of *March*, by which means you may there find any day of the Moneth you desire: Secondly, Next to this is again placed another Æquinoctial Circle divided into 360 degrees. Thirdly, The Ecliptick, containing the 12 signes of the Zodiack, each signe divided into 30 degrees. There is also belonging to this moving Plain a moving Meridian, I call an Index graduated both waies from the Equinoctial, whose length is equal to the Diameter of the Æquinoctial described on the upper moving Plain: The other parts whereof beyond what is graduated serves to cut the Circle of Moneths, the Equinoctial and hour Circles on the lower Plain as occasion is. Lastly, There ought to be also a turning Horizon which may be fitted for any Latitude, having two Indexes thereunto belonging, a longer and a shorter according to the Pattern, to make which Horizons for all Latitudes upon a piece of Past-board,

draw

draw 2 lines perpendicularly croſſing each other in the middle, the length of one of theſe Lines make equal to the Diameter of the upper Sphear, and upon the other Line from the Center mark out the Latitude, by which means there is found three points, now find the Center to thoſe 3 points, and ſtrike an Arch and cut out by that Arch the Horizon required.

PROBL. 1.

How by this Inſtrument to find the Declination place in the Ecliptick, and the Right Aſcenſion of the Sun or any Star.

Firſt, Turn the upper Sphear till Aries ♈, or the 10*th* of *March* ſtands directly againſt *Aries* ♈ in the lower Sphear. Then bring the moving Meridian to the day of the Moneth, if your queſtion be concerning the *Sun*; or if it concern a ſtar, lay the Index upon the Center of the ſtar (whereof you find many with their Names added upon the upper moving ſphear.) This done, obſerve that the Diſtance betwixt the Æquinoctial on the upper plain, and the ſuns place or ſtar; is the ſuns or ſtars Declination; and the point or place cut in the Ecliptick at the ſame time by the Index, ſhews the ſign, degree, and minute, the ſun or ſtar is in;

in; being their place or Longitude in the Ecliptick: And the Degree and Minute then cut in the Equinoctial by the Index, ſhews the ſun or ſtars right Aſcenſion in degrees and Minutes, and the ſame is ſhewn in Hours and Minutes in the outermoſt Hour-Circle of the lower Plain: For Example, the 20*th* of *April* the ſuns Declination is 15 degrees North, his place is 10 *d.* 8 *m.* of ♉ *Taurus*, his Right Aſcenſion 38 *d.* which in time is two hours, 32 minutes.

The 17*th* of *June* the great Dogs declination, 16 *d.* 32 *m.* his place or Longitude 7 degrees of *Cancer* ♋: His Right Aſcenſion 98 *d.* and in time 6 hours, 32 minutes.

PROBL. 2.

To find the Riſing and Setting of the Sun or any Star.

If the ſun or ſtar have North Declination, bring the longeſt Index of the Horizon to 12 hours in *Aries* hour-Circle, which will alſo cut 20 degrees of Longitude in the Equinoctial, the Meridian of *London*: But if the ſun or ſtar hath ſouth Declination, then bring the ſhorteſt Index of the Horizon to the ſame place, and there keep the Horizontal Index faſt, and if you deſire to know its Riſing, turn the upper moving

ving ſphear till the place of the Sun or the ſtar come exactly to touch the Circular edge of the Moving Horizon in the Eaſt Semi-circle of the moving ſphear; or if you deſire its ſetting, bring the Suns place or ſtar to touch the Horizon in the Weſtern Semi-circle: Then turn the moving Index or Meridian to the day of the Moneth, and the ſame Index will there ſhew you the time of the Sun or ſtars riſing or ſetting in *Aries* hour-Circle: For Example, the 20*th* of *April* the Suns place being 10 *d.* 12 *m.* of *Taurus*, riſeth at 4 *h.* 48 *m.* in the morning, and ſets at 7 *h.* 12 *m.* in the Afternoon. Alſo the great Dog the 10*th* of *March* riſeth at one a Clock 52 minutes in the Afternoon, and ſets at 11 hours, or 11 a Clock 8 minutes at night.

PROBL. 3.

To know when any Star comes to the South.

Firſt, Bring the Moving Meridian to 12 hours in *Aries* Hour-Circle, and 20 degrees in the Æquinoctial, the Longitude or Meridian of *London*: keeping that faſt, move the upper ſphear till the ſtar come juſt to the graduated edge of the moving Meridian, and there keep the ſtar faſt, then turn the ſaid moving Meridian to the day of the Moneth,

Month, and in *Aries* Hour-Circle the ſame moving Meridian will ſhew you the time of that ſtars coming to the South: So the great dog the *1ſt.* of *January* comes to the South 11 a Clock at night; and *Orions* left ſhoulder the ſame day at 9 *h.* 37 *m.* *&c.*

To uſe Theakers *Planiſphere inſtead of a Nocturnal, or by it to find the hour of the Night by any Star which is thereon.*

This may be done two ſeveral waies: The firſt is the ſame with that deſcribed (376.) *Prob.* 3. for to know at what hour any ſtar comes to the South muſt neceſſarily ſhew what hour it is when that ſtar comes to the South, and conſequently any other hour may be found by knowing how many hours ſuch a ſtar is paſt the Meridian, or ſhort of it: And therefore,

1. Suppoſe 12 hours in *Aries* hour-circle, and 20 *d.* in the Equinoctial, the Meridian of *London* (which are both one) to be inſtead of the *Flower de Luce*, or North point in a Nocturnal: By help of the Index bring the ſtar on the Inſtrument ſo many hours paſt it, or ſhort of it; as the ſame ſtar is paſt or ſhort of the North-point in Heaven: Keeping the ſtar to that poſition, move the Index to the day of the Moneth, and in *Aries* hour-Circle the ſame Index will ſhew you the hour of the night. 2. Sup-

2, Suppose ♈ to be instead of the *Flower de Luce* or the North point, bring the star in a due position thereunto, and the day of the Moneth shall point exactly to the hour of the night in the third hour-Circle, next below *Aries* hour Circle in the Fixed Plain: So the first of *April* the bright star in the Rump of the great Bears Tail being upon the Meridian, either of these waies it will appear to be ¼ of an hour past 11.

PROBL. 4.

To find at what time the Moon, or any of the Planets come to the South.

First, You must know the Sign and Degree in the Ecliptick which the Moon or Planet is in; and bringing the moving Meridian or Index to 12 hours and 20 degrees the Longitude of *London*, bring the degree which the Moon or Planet is in to the edge of the Meridian, and there keep both fast, and then look in the Circle of Moneths, and right against the day of the Moneth in *Aries* hour-Circle, you will see at what a Clock that Planet comes to the South: But for the Moon you must remember to add 2 minutes for each hour, *vide Theaker's* Book, *pag.* 20.

Those that desire to know more of this Instrument,

Inſtrument, I refer them to the ſame Book of *Theakers*, and ſo paſs on to ſhew how the moſt of theſe Queſtions of the Sphear may be found.

PROBL. 5.

As an Appendix to theſe things, Concerning the Moon, I ſhall ſhew you how to find the Moons place.

Multiply the Moons age by 2, and divide the Product by 5, the Quotient will ſhew how many Signs, and the Remainder or Fraction, ſo many times 6 degrees as the Moon is gone from that ſign and degree where the Sun is at that preſent: Or more readily by this following Table.

The

The Dayes of the Moons Age.

D.	S.	D.	M.
1	0	13	11
2	0	26	21
3	1	9	32
4	1	22	42
5	2	5	53
6	2	19	3
7	3	2	14
8	3	15	25
9	3	28	35
10	4	11	46
11	4	24	56
12	5	8	7
13	5	21	18
14	6	4	28
15	6	17	39
16	7	00	49
17	7	14	—
18	7	27	11
19	8	10	21
20	8	23	32
21	9	00	42
22	9	19	53
23	10	3	3
24	10	16	14
25	10	29	25
26	11	12	35
27	11	29	46
28	00	8	56
29	00	22	7
30			

The Hours of the Moons Age.

H.	S.	D.	M.
1	—	0	33
2	—	1	6
3	—	1	39
4	—	2	12
5	—	2	45
6	—	3	18
7	—	3	51
8	—	4	24
9	—	4	56
10	—	5	29
11	—	6	2
12	—	6	35
13	—	7	8
14	—	7	41
15	—	8	14
16	—	8	47
17	—	9	20
18	—	9	53
19	—	10	26
20	—	10	59
21	—	11	32
22	—	12	5
23	—	12	38
24	—	13	11

The use of the Table by an Example.

As suppose the 19*th* day of *June* 1675, at Noon, you would know what sign the Moon is in: The change was the 12*th* day, at 8 hours at night; therefore the 19*th* day at Noon the Moon is 6 dayes, 16 hours old.

The place of the Sun and Moon at the change was 3 signs, 1 degree, 0 *m.* the Moons motion for 6 dayes by the Table is 2 signs, 19 *d.* 3 *m.* and for 16 hours 8 *d.* 47 *m.* the sum is 5 *s.* 28 *d.* 50 *m.* that is in 28*d.* 50*m.* of *Virgo*, which is the 5 sign from *Aries*: The like may be done at any other time. So much shall suffice to have spoken concerning the Moon.

SECT. 3.

By the Quadrant.

I shall first give you a Description of the Lines thereupon, and then shew you the uses thereof: The double Arch next the Center is only for Ornament: The next Arch is the Æquinoctial, and the next to that represents the Tropique; betwixt which two Arches is the Line of Declination upon the side of the Quadrant, containing 23 *d* 30 *m.* and within are the Azimuth or hour-lines: Also Crosswise betwixt the

the Æquinoctial and Tropique is the Ecliptick, containing three of the signs, each sign divided into 30 *d.* or at least supposed to be so: Next to the Ecliptick, and joyning with it in the Æquinoctial is the Horizon, divided, or supposed to be divided into 40 *d.* and ending or crossing the Tropick in 33 *d.* 9 *m.* Next below the Tropick is the Circle of Months, which you may know by the Letters prefixt: The next Arch below these is the Quadrat, or that which serves instead thereof, the line of right shadows, and contrary shadows numbred from 1 to 10, and so back again to 1.

And the lowest of all is the Limb of the Quadrant divided into 90 degrees, and each degree into halfs. For the use thereof,

1. The Thread laid on the day of the Moneth, and the Bead just to touch the hour-line of 12: at the same time the thread will shew in the Limb of the Quadrant the Meridian Altitude of the sun, and moving the Bead from one hour to another, will shew the suns Altitude at any hour whereon the Bead resteth, and the Bead amongst the hour-lines will shew the hour of the day.

2. *How to find the Hour of the Night by some principal Stars set upon the Quadrant.*

How to place any star upon the Quadrant,

First,

Firſt, You muſt know the ſtars Declination from the Æquator, and his Right Aſcenſion from the next Æquinoctial Point: As the Declination of the Wing of *Pegaſus*, being 13 *d.* 17 *m.* His right Aſcenſion 358 *d.* 34 *m.* from the firſt point of *Aries*, or 1 *d.* 26 *m.* ſhort of it: If you draw an occult parallel through 13 *d.* 17 *m.* And then lay a Ruler to the Center A, and 1 *d.* 26 *m.* in the Quadrant, the point where the Ruler croſſeth the Parallel, ſhall be the place for the Wing of *Pegaſus*, and ſo for the reſt.

Stars.	R. Aſcenſion.	Declination.
Arcturus	30 *d.* 7 *m.*	21 *d.* 6 *m.*
Lyons Heart	32 *d.* 28 *m.*	13 *d.* 42 *m.*
Bulls Eye	64 *d.* 18 *m.*	15 *d.* 46 *m.*
Aquila	66 *d.* 26 *m.*	8 *d.* 3 *m.*

To find the Hour of the Night by theſe Stars.

Firſt, put the Bead to the ſtar which you intend to obſerve, take his Altitude, and note how many hours he is from the Meridian, then out of the right Aſcenſion of the ſtar take the right Aſcenſion of the ſun converted into hours, and mark the Difference; for this difference being added to the obſerved hour of the ſtar from the Meridian, ſhall ſhew how many hours the *Sun* is gone from the Meridian, which in effect is the hour of the Night: As for Example, the 9*th* of *July* the *Sun* being then in 26 *d.* ♋

of

of *Cancer*, I ſet the Bead to *Oculus Tauri*, or the Bulls Eye, and obſerving his Altitude, ſhould find him to be in the Eaſt about 12 degrees high, and the Bead to fall on the hour-line of 6 before noon, which is 18 hours paſt the Meridian, (or paſt 12 at Noon.) The hour of the Night would be better than a Quarter paſt two of the Clock in the morning. For 118 *d.* the Right Aſcenſion of the *Sun* converted into time, make 7 *h.* 52 *m.* this taken out of 4 *h.* 15 *m.* the Right Aſcenſion of *Oculus Tauri* adding a whole Circle (or 24 hours) for otherwiſe there could be no ſubſtraction, the Difference will be 20 *h.* 23 *m.* and this being added to 18 hours, which was the obſerved Diſtance of *Oculus Tauri* from the Meridian, ſhews that the *Sun* (abating 24 hours for the whole Circle) is 14 *h.* 23 *m.* paſt the Meridian, and therefore 23 *m.* paſt two of the Clock in the morning.

3. At what hour the Bead toucheth the Line of Declination, that is, the hour of the *Suns* riſing or ſetting: And bring the Bead to the Horizon, and the degrees cut in the Limb is the Aſcenſional Difference.

4. The degrees cut by the Bead in the Line of Declination, ſhews the *Suns* Declination for that Day.

5. The Bead brought to the Ecliptick ſhews the *Suns* place.

5. Brought

5. Brought to the Horizon ſhews the Amplitude, which is alwaies North when the *Sun* is in Northern ſigns, and South when the *Sun* is in Southern.

6. The Bead ſhews the Azimuth, the thread being laid to the *Suns* Altitude.

7. The thread laid to the *Suns* place, it ſhews in the Limb of the Quadrant his right Aſcenſion.

8. Moving the Thread backwards from the Line of Declination towards the hour-line of 12: It ſhews in the Limb the degrees of the *Suns* Depreſſion under the Horizon at each hour, and conſequently when it is 18 degrees under the Horizon it ſhews the beginning of day break, and end of twilight.

Thus you ſee all theſe Aſtronomical Queſtions are eaſily and ſpeedily reſolved by the Quadrant, but obſerve that theſe Quadrants do ſerve only for ſome particular Latitudes.

As an Appendix I ſhall add the ingenious uſe of the Quadrat upon the Quadrant, and ſo conclude this Point: This which is called the Quadrat is ſupplyed in a Line drawn next above the Limb of the Quadrant numbred from 1 to 10 in the middle, and ſo back again to 1, (as in the Deſcription,) whereof the former part are called

Right ſhadows, and the latter to the right hand are called contrary ſhadows, and by theſe you may find an Height by a known Diſtance, or a Diſtance by a known Height. And for performing hereof take theſe Rules.

1. If the thread fall in a right ſhadow, As 100 is to the parts on which the Thread falleth, ſo is the Diſtance to the Height required: Or, As the parts cut by the Thread is to 100, ſo is the Height to the Diſtance required.

2. If the Thread fall in a contrary ſhadow, As the Parts cut by the Thread are to 100, ſo is the known diſtance to the Height required: Or, As 100 is to the parts cut by the Thread, ſo is the known Height to the diſtance required. To make this more plain, If the thred fall on 100 parts, which is denoted by 10 in the Quadrant, or on 45 degrees in the Quadrant; the Height is equal to the diſtance above the Level of your Eye: If it fall on 50 parts of Right ſhadow, the Height is half the diſtance, if it fall on 25, it is $\frac{3}{4}$ 3 quarters of the diſtance: But if the Thread fall on 50 parts of contrary ſhadow, the Height is double to the diſtance; If on 25, it is 4 times the diſtance, and the like of the reſt.

CHAP.

CHAP. VIII.

BUT to return to my propoſed Method, having ſhewn you how theſe Sphærical Problems or Queſtions of the Sphear may be eaſily and readily reſolved by thoſe Artificial Projections of the *Globe*, the *Planiſphear*, and *Quadrant*, and by way of Digreſſion, having added the other uſes alſo of theſe Inſtruments, I come now in the laſt place to ſhew how theſe queſtions may be reſolved by *Projection of the Sphear in plano.* And to do this,

1. I ſhall ſhew you how to *Project the Sphear in plano.*

2. How theſe Sphærical Triangles do naturally lye in the Sphear it ſelf: Thus Projected.

3. How the Sides and Angles of theſe ſphærical Triangles may be meaſured in the Projection by a Line of Chords only, whereby theſe queſtions are moſt ingeniouſly reſolved.

SECT. 1.

How to Project the Sphear in plano upon the Plain of the Meridian.

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Norwood in his *Seam*
Brown, *pag.* 40. Alſo
genioufly: But that
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uſed by *Gemma Friſi*
ſo find in *Brown*, *p.* 1
metrical Seaman, *pag.*
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Firſt then, Take 6
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tereographick Projection,
Friſius, which you will al-
p. 142, and in *Philips* Geo-
pag. 75. which is the way
the whole World: But
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hat Projection of Mr. *Ley-*
etrical Exerciſes *pag.* 122.
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The Projection.
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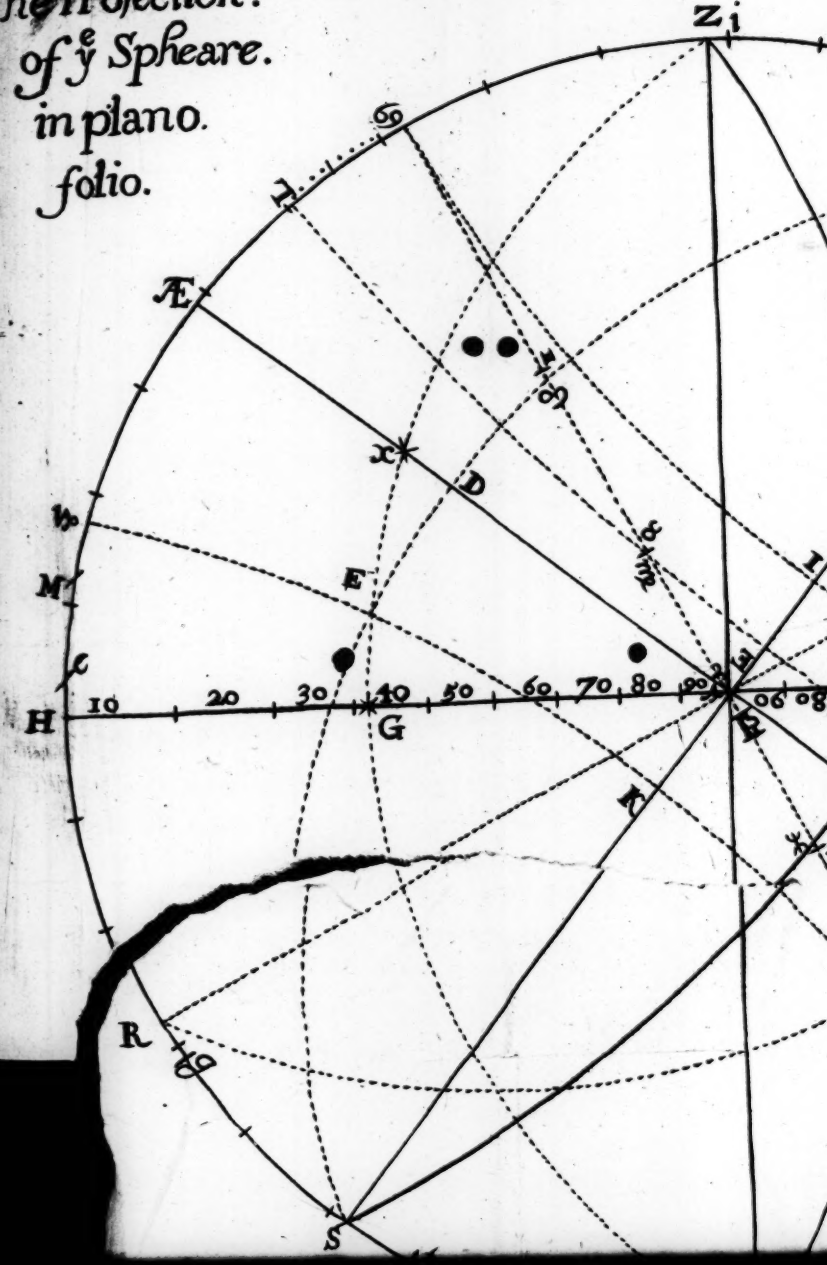

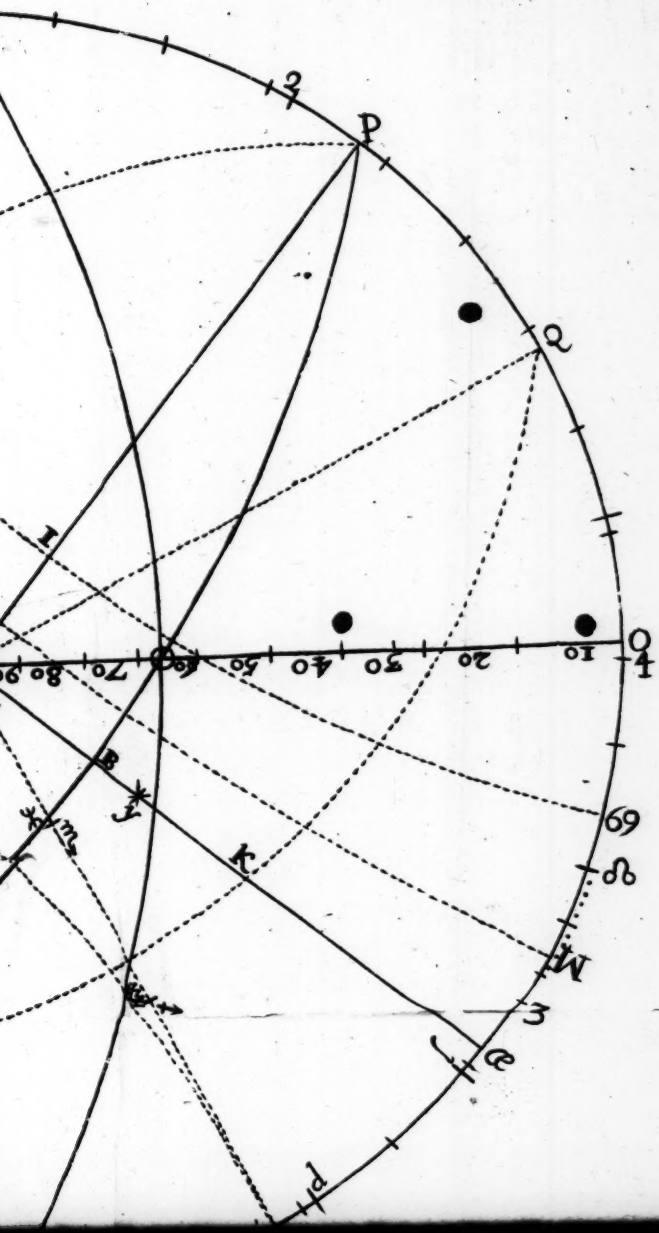
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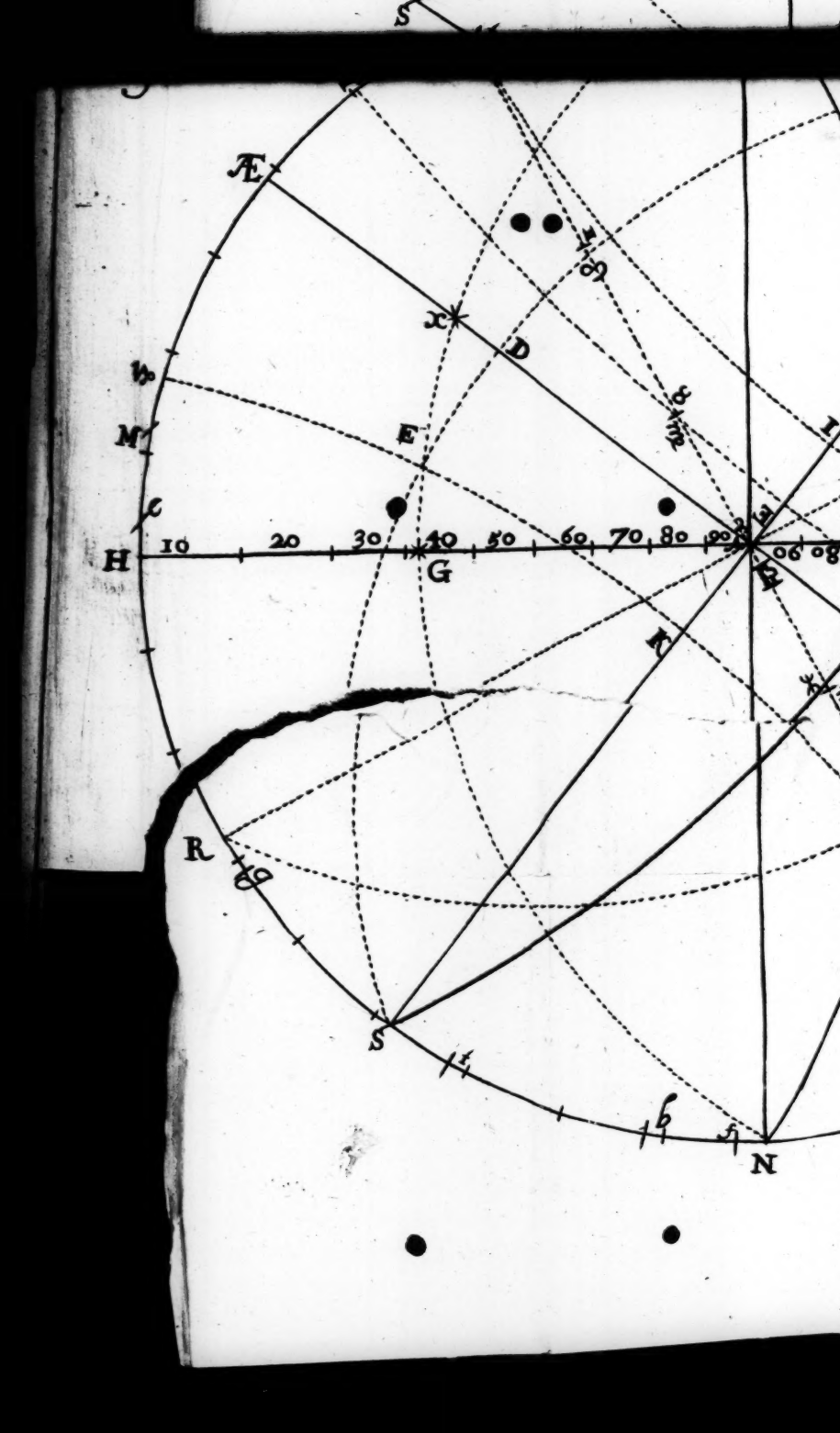

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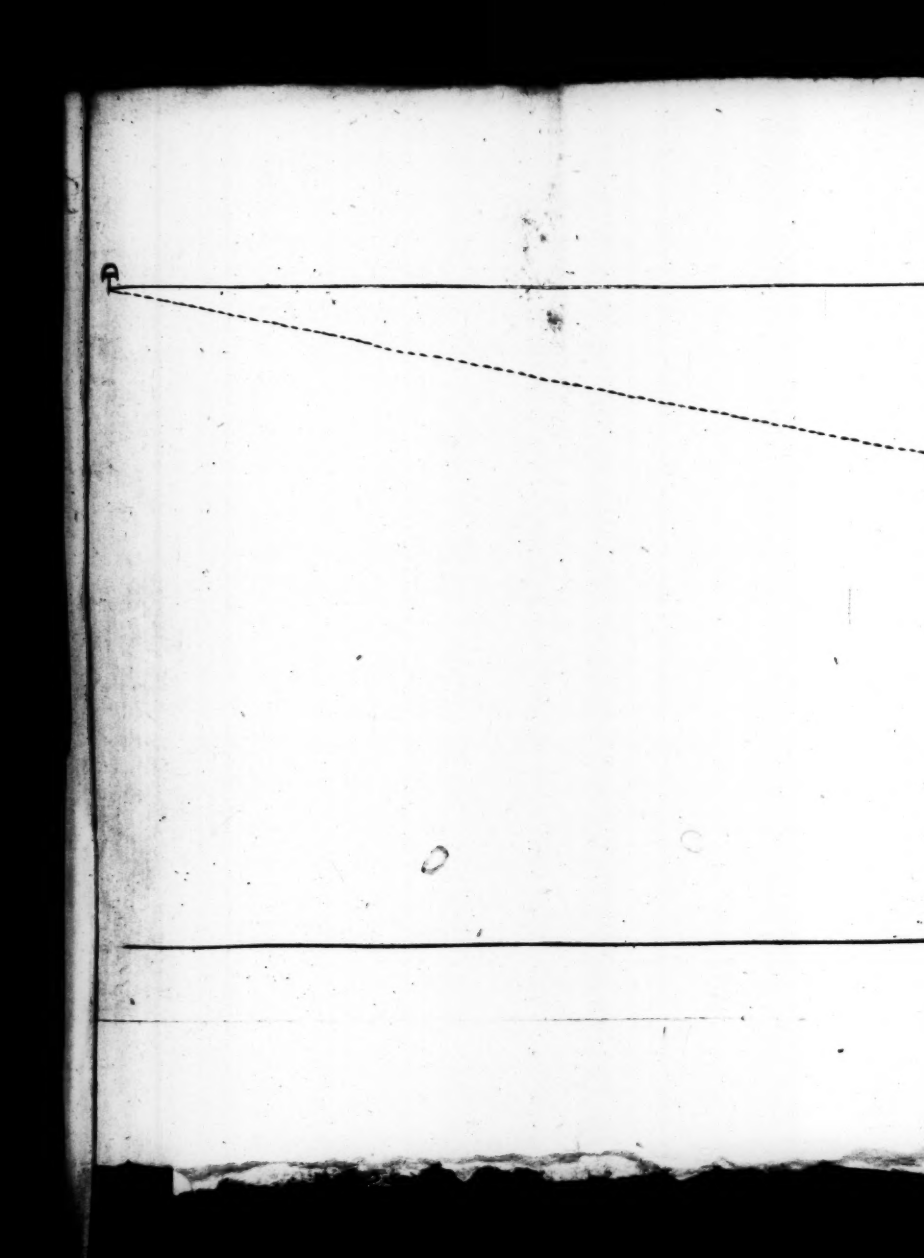

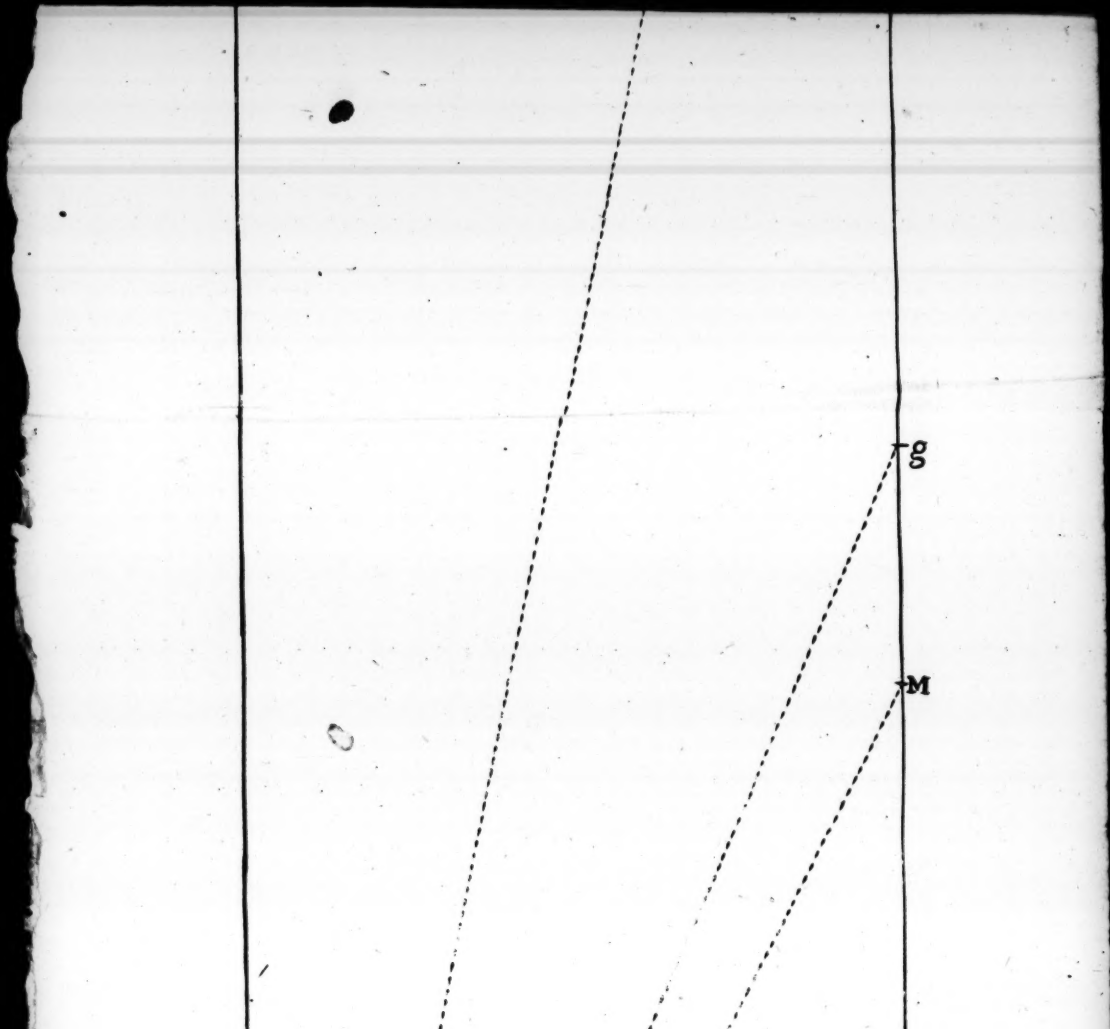
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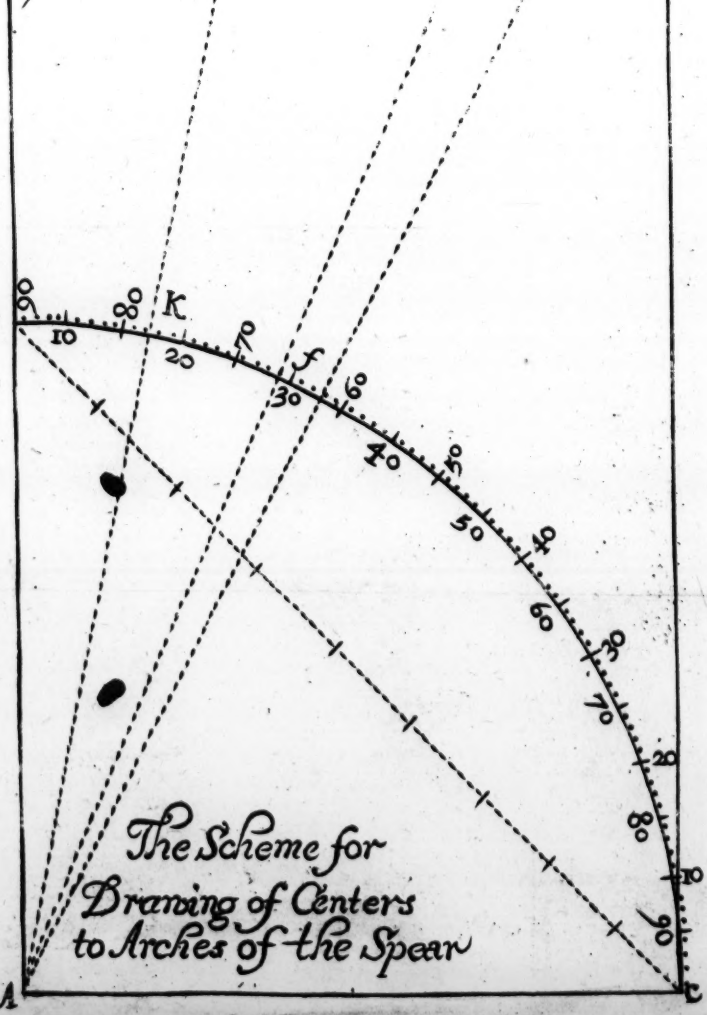


The Scheme for
Drawing of Centers
to Arches of the Spear

Diameters HAO, and ZAN, the former repreſenting the Horizon, the latter the Zenith and Nadir, or the prime Vertical.

Secondly, (Becauſe the Latitude of the place for which you draw your projection is 51 *d.* 30 *m.*) Take 51 *d.* 30 *m.* from your Line of Chords, and ſet them upon the Meridian from O to P, and from H to S, and draw the Line PAS, the Axis of the World P ſignifying the North Pole, and S the South Pole: Alſo with the ſame diſtance ſet off 51 *d.* 30 *m.* from Z to Æ, and from N to æ, and draw the line Æ Aæ for the Equinoctial.

Thirdly, Take 23 *d.* 30 *m.* the *Suns* greateſt Declination, and alſo the Diſtance of the 2 Tropicks from the Equinoctial, and ſet them upon the Meridian from Æ to ♋, and from Æ to ♑; and alſo from æ to ♋, and from æ to ♑: This done, draw a right Line ♋ A ♑ between the two Tropicks, touching the Tropick of *Cancer* above the Horizon, and the Tropick of *Capricorn* below the Horizon, and this is the Ecliptick: Then take 23 *d.* 30 *m.* out of your Line of Chords, and ſet them from P to Q, and from S to R, and draw the Line QR the Axis or Poles of the Ecliptick. Now to divide the Ecliptick into ſigns and degrees, take 60 *d.* from your Line of Chords, and ſet them from Q to 1, and from Q to 3: Alſo ſet the ſame diſtance

from ♋ to 2, and from ♍ to 4; this done, lay a Ruler to the Pole R, and the figure 1, it will cut the Ecliptick in the point ♊ and ♌: The Ruler laid to R and 2 will cut it in the points ♀ and ♏, and laid to R and 4 in ♏ and ♐; and laid to R and 3 in ♐ and ♒. And if you divide every of the ſpaces ♋, 1, 12, 2 Q, Q4, 43, and 3 ♍ into 3 equal parts, each part will contain 10 degrees, and a Ruler laid to each of them, and the point R ſhall give you the points upon the Ecliptick anſwering to the 10 degrees of every ſign, and in the ſame manner you may put on every degree. Thus you have drawn the Meridian Horizon, the prime Vertical, the Axis of the World, and the Æquinoctial, the Axis of the Ecliptick, and the Ecliptick, all which are ſtreight lines: Now we come to draw thoſe which are Circular, or Arches of Circles; and before you can draw theſe, the Centers of thoſe Circles muſt be found out: Now I ſhall ſhew you two waies of finding out theſe Centers: One of theſe is Mr. *Phillips* in his *Geometrical Seaman*, on this manner, divide the Meridian into degrees in 4 Quadrants, and keeping one end of your Ruler fixed at the Point at Z: Lay the other end to the ſeveral degrees in the lower Semi-circle HNO, ſo you ſhall divide the Diameter HO into 9 parts, which are half-

half-Tangents. In the ſame manner, if you keep one end of your Ruler fixed at the point H, and lay the other end to the ſeveral degrees of the Semi-Circle ZON. You may divide the Diameter ZN into 9 unequal parts, which are half-Tangents. Now having thus divided the Diameters, they muſt guide you in drawing of the Meridians, and Parallels which are all parts of perfect Circles: And you may find their Centers by theſe three points. Firſt, For the Meridians, they all concur in both the Poles, and their third point is their correſpondent degree in the oppoſite Diameter, and for the Parallels, two of their points are their degrees in the outward Circle, and their third point is their correſpondent degree in the oppoſite Diameter: Further you muſt know that the Centers do all lye in the Diameter Lines, extended beyond the Circle, (if you ſeek for the Center of a Meridian exceeding 45 degrees) for the Diameter being divided into half-Tangents, if for every degree you account 2 beginning from the Center A, ſo you ſhall have the Centers of the Meridians, and if you ſet one foot of your Compaſſes in that Center, and open the other to the Point Z or N, it will paſs through the Correſpondent degree in the Diameter H. O. As for Example, If you would draw the Meridian

ZGN, which cuts the Diameter in 32 degrees, HG, count from the Center towards O, double ſo many degrees as is betwixt H and G, which is 64 degrees, and that is the Center to the Meridian, Z. G. N. *viz.* W. The like is to be done for finding the Centers to the Parallels, as you may eaſily apprehend: But if any of theſe Centers fall ſo far without the Circle, that your Compaſſes will not reach them, there is an Inſtrument call'd a Bow, which by help of one or more ſcrews may be extended to touch any three points which lye near in a ſtraight Line; or a Ruler bridled may do the ſame if it bend equally.

This way of Mr. *Phillips* finding the Centers I cannot but applaud as very ingenious, but do prefer that of Mr. *Leyburns*, which is thus: Upon a long piece of ſtiff paper deſcribe a Quadrant as ABC, and upon the point C erect an exact perpendicular. Let the Quadrant be divided into 90 degrees, and conſider how many degrees the Arch of the Circle (whoſe Center you ſeek) is diſtant from the Meridian or outward Circle, and a Ruler laid from the Center of the Quadrant by ſo many degrees in the Limb thereof to cut the perpendicular, where the Ruler cuts the perpendicular make a prick or mark, and the diſtance betwixt the ſaid mark and the Center of the Quadrant A

ſhall

ſhall be the Semidiameter of the Circle ſought: To make this plain by theſe following Examples, Firſt, for the Tropicks, whoſe Diſtance from the Equinoctial is 23 *d.* 30 *m.* lay the Ruler to the Center of the Quadrant A, ſo as it cut 23 *d.* 30 *m.* from the point B in the Limb at *f*, and cuts the perpendicular at *g*, the Diſtance A *g*, or the Line A *f g* taken in your Compaſſes, and ſetting one foot upon the Polar Line extended till the other foot juſt reach the Equinoctial at Æ; draw an Arch of a Circle, and it ſhall be one of the Tropicks, as ♋ I ♋ the Tropick of *Cancer*, and ♑ *k* ♑ the Tropick of *Capricorn*: The like may be done by any other parallel of Declination, or Latitude, whether they fall nearer or further off the Equinoctial.

Another Example take in the Hour-Circle PBS, which is diſtant from the Meridian 62 *d.* 46 *m.* wherefore take 62 *d.* 46 *m.* from your Line of Chords, and ſet them from *a* to *b*, then laying a Ruler from P to *b*, it will cut the Equinoctial in B, through which point the hour-Circle of 62 *d.* 46 *m.* muſt paſs. To find the Center belonging to this hour-Circle, lay your Ruler to the Center of the Quadrant at A, and let it paſs by 62 *d.* 46 *m.* in the Limb: From the point C it will cut the perpendicular in M, therefore taking the Diſtance in A in your

Compaſſes, and ſetting one foot in the Equinoctial extended, and with the other juſt to reach P or S, ſtrike an Arch of a Circle, and it will cut PBS the Hour-Circle of 62 *d.* 46 *m.* The like may be done in any other hour-Circle, or Azimuth-Circle, Parallels of Declination, Circles of Altitude, or any other whatſoever; only obſerve, that the Centers of all hour-Circles fall in the Equinoctial Line Æ A *e* extended, the Centers of Azimuth-Circles in the Horizon HAO extended, where need is: The Centers of the Tropicks, and parallels in the Axis of the World PAS, and the Centers of the Circles of Altitude fall in the prime Vertical Circle ZAN.

Laſtly, As every Circle in the Projection hath its proper Center, ſo hath it alſo its proper Pole. For the finding whereof, note that the Pole of a great Circle is 90 degrees, or a Quadrant diſtant from the Circle it ſelf upon that Line which cutteth the Circle at Right Angles: Thus the Poles of all the Hour-Circles are upon the Equinoctial: The Poles of all the Azimuth Circles upon the Horizon, *&c.* Now if you would know the Pole of the Hour-Circle PDS, lay a Ruler upon P and D, and it will cut the Meridian-Circle in *e*: Then take 90 degrees from your Line of Chords, and ſet them from *e* to *f*, a Ruler laid from P

to

to *f*, will cut the Equinoctial in Y, ſo is Y the Pole of the Hour-Circle PDS, the ſame Rule is to be obſerved in finding the Poles of the Azimuth-Circles: As if you would find the Pole of ZGN, lay a Ruler upon Z and G, it will cut the Meridian-Circle in *g*, then ſet 90 degrees of your Chord from *g* to *d*, ſo a Ruler laid from Z and *d* will cut the Horizon HAO in the point ⊙, which is the Pole of the Azimuth-Circle ZGN; and by the ſame manner of work you may find the Poles of all the reſt. As of PDS is Y, PCS, V; PAS, Æ or *e*, PBS. ZGN, ⊙, ZAN, H or O; ZFN, T; Z⊙N, G; HAO, Z and N; Æ A *e*, P and *S*; ♋ A ♑, Q and R; And thus much ſhall ſerve to ſhew you how to Project the Sphear *in plano*, and how to find the Centers and Poles of all the Circles.

SECT. 2.

In the next place I come to ſhew you how the forementioned Queſtions of the Sphear may be ſpeedily and exactly reſolved by the Reſolution of ſeveral Spherical Triangles which naturally lye within the Sphear being thus projected: And of theſe in this following order: Having before ſhewn you how to meaſure the ſide or Angle of any Sphærical Triangle, *pag.* 344, 345.

PROP.

PROP. 1.

The Diſtance of the Sun from the neareſt Æquinoctial Point (either Aries *or* Libra*) given, To find his Declination.*

In the Projection this Propoſition is to be reſolved upon the Triangle A *k* ♒; wherein you have given, 1. The Side A ♒ 59 *d.* the diſtance of the Sun from the neareſt Æquinoctial point *Libra.* 2. The Angle ♒ A *k*, 23 *d.* 30 *m.* which is the Angle that the Ecliptick makes with the Equinoctial, and is alwaies equal to the greateſt Declination of the Sun, and you are to find the ſide *k* ♒, which to do by the Projection.

Lay a Ruler upon the Pole of the Circle R ♒ Q (which is at ♉) and the point *k*, it will cut the Meridian in the point *l*: Alſo lay a Ruler from ♉ to ♒, it will cut the Meridian in the point ♍, ſo the diſtance *l* ♍ being meaſured upon your Line of Chords, will contain 20 degrees, the Suns Declination, he being in 29 *d* of ♒.

PROP. 2.

To find the Latitude, or the Side PO of the Triangle P ☉ O.

You need do no more when you are to meaſure any ſide of a Triangle, which lies in

in the Meridian it ſelf, than to take the ſame ſide in your Compaſſes and meaſure it upon your Line of Chords, as in this Propoſition. Take in your Compaſſes the Diſtance from O to P, and meaſure it upon your Line of Chords, and it will be found to contain 51 *d.* 30 *m.* the Latitude required.

PROP. 3.

The Latitude of the place, and Declination of the Sun given, To find his Amplitude.

This Propoſition may be reſolved by two ſeveral Triangles, one is P⊙O, the other A⊙B. In the firſt you have given the ſide PO the Latitude, and P⊙ the Complement of the Suns declination, and you are to find ⊙O. To do this by the Triangle, A⊙B, lay a Ruler upon Z, (which is one of the Poles of the Horizon) and to the point ⊙ it will cut the Meridian Circle in *d*, and laid from Z to A, it will cut the Nadir point in N; ſo the diſtance N *d* will be found 33 *d.* 20 *m.* The Amplitude of the Suns Riſing or Setting from the true Eaſt or Weſt points of the Horizon. In the Triangle A⊙B, you have given ⊙B the Suns Declination 20 *d.* The Angle ⊙AB, the Complement of the Latitude 38 *d.* 30 *m.* and you are to find the ſide ⊙A as above.

For

For finding the ſide ☉O in the Triangle P☉O, lay a Ruler to Z, the Pole of the Horizon, and the point ☉, and it will cut the Meridian Circle in D, ſo the diſtance *d*O meaſured upon your Line of Chords, will contain 56 *d*. 40 *m*. which is the Amplitude of the Suns Riſing or Setting from the North point of the Horizon.

PROP. 4.

The Suns greateſt Declination, with his Diſtance from the next Equinoctial point being given, To find his Right Aſcenſion.

The Triangle A *k* ♈ reſolves this propoſition, wherein you have given, 1. The ſide A ♈ the diſtance of the Sun from *Libra*. 2. The Angle ♈ A *k* the Suns greateſt Declination, and the right Angle at ♈, and you are to find A *k* the ſide of the Triangle. Lay a Ruler to P and *k*, it will cut the Meridian in *s*, and lay it again to P and A, it cuts the ſame in S, ſo the Diſtance *S s* will be found 56 *d*. 50 *m*. the Suns Right Aſcenſion.

PROP. 5.

The Latitude and Declination given, To find the Aſcenſional Difference.

This is reſolved in the Triangle A☉B, wherein you have given, 1. The ſide ☉B, the

the Suns Declination 20 *d.* 2. The Angle ⊙AB, the Complement of the Latitude 38 *d.* 30 *m.* and the Right Angle at B, and you are to find the ſide AB. Lay a Ruler to P, the Pole of the World (and alſo of the Equinoctial) and the point B, it will cut the Meridian Circle in the point *b*, the diſtance *b. s.* meaſured, will be 27 *d.* 14 *m.* the Aſcenſional difference, which is ſo much as the Sun Riſeth or Setteth before or after ſix a Clock.

PROP. 6.

The Latitude, Declination, and Altitude of the Sun given, To find the Suns Azimuth either from the Eaſt, North, or South points of the Horizon.

That which is moſt difficult and intricate to perform by Numbers, is by Projection effected with the ſame eaſe as any of the reſt: As in this propoſition it is the Angle EZP, in the Triangle ZEP, which is here required: Lay a Ruler upon Z to the point G upon the Horizon, the Ruler thus laid, will cut the Meridian in G, ſo the diſtance *g.* O. meaſured upon the Line of Chords, will be 146 *d.* which is the Suns Azimuth from O the North part of the Meridian, and from *g* to H 34 *d.* which is the Azimuth from H the South-part of the Meridian,

and

and from *g* to N 56 *d.* which is the Azimuth from A, the Eaſt and Weſt point of the Horizon.

PROP. 7.

The Latitude, Declination, and Altitude of the Sun given, to find the hour of the day.

This Propoſition is performed by reſolving the Triangle Z P *a*, where you are to find the Angle Z P *a* : Wherefore lay a Ruler from the point P, to the point *a*, and it will cut the Meridian in *t*, ſo the Arch *t* Æ will be found 95 *d.* 52 *m.* which is the hour from the Meridian: The Arch *t a* being meaſured, will contain 84 *d.* 8 *m.* which is the hour from midnight : Alſo the Arch *t* S being meaſured, will contain 5 *d.* 52 *m.* the hour from ſix.

This Triangle Z P *a* is compoſed of Z P an Arch of the Meridian Z, *a* an Arch of an Azimuth Circle, and of P *a* the Arch of an hour-Circle.

PROP. 8.

The Declination, Altitude, and Azimuth of the Sun given, to find the hour of the day.

The Triangle ZEP in the projection reſolves this Problem, wherein there is given

1. EP,

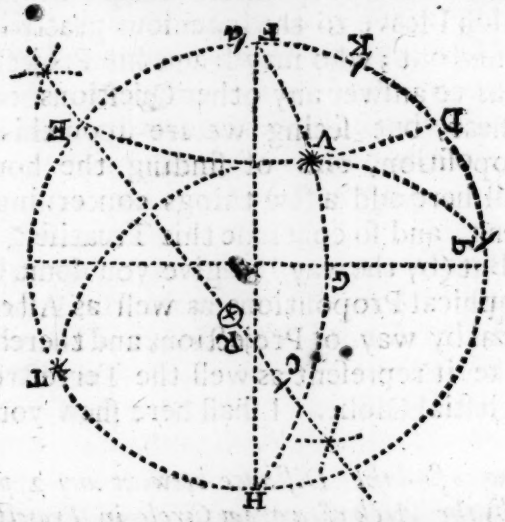
1. EP, the complement of the Suns Declination 70 *d.* South, 2. The side EZ the complement of the Suns Altitude 78 *d.* And 3. The Angle EZP the Suns Azimuth from the North. The Angle ZPE is to be found to answer this proposition: Therefore lay a Ruler to the Pole P, and upon the point D in the Æquinoctial, and it will cut the Meridian in *e*. The distance *e* Æ being measured, will give you 35 *d.* 36 *m.* the hours from Noon which in time is 2 *h.* 22 *m.* Many more propositions might be added, which I leave to the ingenious practitioner to find out; who may frame his Projections so as to answer any other Questions of the Sphear, but seeing we are upon this last Proposition, *viz.* of finding the hour, I shall here add a few things concerning Dialling, and so conclude this Treatise.

But (by the way) I'll give you some Geographical Propositions as well as Astronomical by way of Projection, and thereby to make it represent as well the Terrestrial as Cœlestial Globe. I shall here shew you,

How to find the Distance betwixt any 2 places by the Arch of a great Circle in Projection, howsoever distant from one another.

1. Draw a Circle, which may be supposed

ſed to be the firſt Meridian, (which is reckoned from the Iſlands of Azores) croſs this Circle through the Center with a Diameter, which is to be ſuppoſed the Equinoctial; If the two places differ only in Longitude, the Diſtance is to be meaſured upon the Equinoctial, or ſome other parallel: If they differ in Latitude only they are to be meaſured upon a Meridian; If they differ both in Longitude and Latitude, the Diſtance muſt be meaſured by drawing an Arch of a great Circle to paſs through



both the places: And to meaſure any of thoſe Diſtances, you muſt find the Pole to the great Circle which paſſes through both places:

places: How to do this, you have it elsewhere taught *pag.* 345: Then laying a Ruler to the said Pole, and first to one of the places, and afterwards to the other, and marking at each time where the Ruler cuts the Meridian or outward Circle: The same distance taken in your Compasses, and measured upon your Line of Chords, gives in Degrees and Minutes the Distance betwixt the two places, which you know being multiplyed by 20, brings it into Leagues, or by 60 into Miles: I shall only give one Example for all: Suppose you would know the Distance betwixt the *Cape of Good Hope*, lying in the Latitude of 40 *d.* South and in Longitude 50 *d.* and *Malibrigo* lying in 26 *d.* of North Latitude, and in 180 *d.* of Longitude you will find their distance to be 140 *d.* or 8400 miles by Projection thus, (H. T. E. F. K. D.)

First, draw the Circle, and from the same Line of Chords by which you draw the Circle take 26 *d.* for the Latitude of *Malibrigo*, and set it from *a* upwards, and because the Longitude is 180 *d.* there you must place it in the Meridian at T. which signifies *Malibrigo*: Now to place the *Cape of Good Hope* according to its Latitude and Longitude, do thus: Draw a Parallel of Latitude of 40 *d. viz.* DE, and a Meridian of 50 *d.* (H. G. F.) for the Longitude

gitude where theſe 2 interſect each other, is the place you deſign (the *Cape of good Hope*) there make a mark at V, and with the ſame Radius wherewith you drew the firſt Circle, find a Center which will cut T and V, which you may eaſily do by drawing a Line to paſs exactly in the middle betwixt T and V, when that is done, you muſt find the Pole of the ſame Circle at R, and laying a Ruler from the Pole R to V, it will cut the Meridian in K. Laſtly, Meaſure the Diſtance T K, and that gives you the degrees that one of the places is diſtant from the other.

Thoſe that deſire to be informed more at large concerning finding theſe Diſtances, I refer them to Mr. *Leyburn*, his Eighth Geometrical Exerciſe, *pag.* 131.

What is here ſaid concerning finding theſe Diſtances, the Ingenious may apply to Sailing by the Arch of a great Circle; but I ſhall leave that to the Readers own Study and Improvement.

To know how many Miles any place is diſtant from you, in whoſe Zenith the Sun, or any Star is at the ſame time.

Take the Altitude of the Sun, which ſuppoſe 53 *d.* from your Zenith; and with Mr. *Norwood* account 69 Miles in a degree, multiply 53 by 69, makes 3657 and ſo many miles

Miles it is between the place where I now am, and the place in whoſe Zenith the Sun was at that time, and by this means you may know how far it is to any place over, which the Sun or Star is at any time.

CHAP. IX.

Of Dialling.

THE whole and compleat Art of *Dialling* is a ſubject ſufficient for a Treatiſe alone, and you will find ſeveral Authors have writ whole Books concerning the ſame, I ſhall confine my ſelf within the compaſs of theſe few leaves which are left me; but what ſhall be ſaid I hope will be plain and perſpicuous to any ordinary Capacity: Brevity, and withal perſpicuity being the Mark I aim at all along in this little Treatiſe.

There are many Denominations and Diſtinctions given to Dials, as *Horizontal*, *Vertical*, *Polar*, *Equinoctial*; *Erect*, *Direct*, *Inclining*, *Declining*, and *Reclining*, every way Eaſt, Weſt, North, and South, and many more, both Fixed and Pendulant: Plain and Globular within doors and without: It is not my intention to ſpeak to theſe particularly, but ſhall only ſhew you how to make an *Horizontal*, and an *Erect*, *Direct South Dyal*, which are moſt uſual, and moſt uſeful.

SECT.

SECT. 1.

To draw an Horizontal Dyal by the Globe.

I have ſhewed you *pag.* 374, how to make the Globe it ſelf a Dyal, and by conſequence any round Ball: I ſhall now ſhew you how to make an *Horizantal Dyal* by the Globe.

Firſt then, Prepare your flat Dyal ground parallel to the Horizon, and thereupon draw a Meridian, which muſt repreſent the direct North and South, and croſs the ſame at right Angles, which muſt repreſent the Eaſt and Weſt. The firſt of which Lines is the 12 a Clock Line, the latter ſhews 6 a Clock both Morning and Evening; then making the Interſection of theſe Lines the Center, deſcribe a Circle on the Dyal-ground to what diſtance you pleaſe, and divide the ſame into 360 *d.* (as all other Circles are) and ſubdivide the ſame into as many leſſer parts as you can: Being thus prepared, you are to find the diſtances of the Hour-Lines in this Circle for any Latitude of place, which to do, let the Globe be ſet to the Latitude deſigned, and then make choice of ſome of the great Circles in the Globe that paſſe through the Poles of the World, as for Example (the Æquinoctial colure) and apply the ſame to the Brazen Meridian, in which ſcituation it repreſents the Subſtile or 12 a Clock Line, or Meridian

ridian Line (call it which you will) then turning about the Globe towards the Weſt, till 15 degrees of the Æquator have paſſed through the Brazen Meridian: You muſt mark the Degree of the Horizon which the ſame Æquinoctial colure croſſes, for that point will ſhew the Diſtance of 1 and 11 a Clock from the Meridian or 12 a Clock-Line: Then turning again the Globe forwards till other 15 degrees of the Equator do paſs the Meridian, the ſame colure will point out the diſtance in the Horizon of the hours 10 and 2 from the hour of 12, and in the ſame manner you may find out the diſtances of all the reſt of the hour-lines from 12 in the Horizon; but note that the beginning of theſe diſtances muſt be accounted from that part of the Horizon on which the Pole is elevated; Now theſe Diſtances of the hours being thus noted in the Horizon of the Globe, you muſt afterwards tranſlate them into your plain allotted for your Dial-ground, reckoning in the circumference of it ſo many degrees to each hour as are anſwerable to thoſe pointed out by the colure in the Horizon: Laſtly, having thus done, the Gnomon or Style muſt be erected, where note that the edge of the Gnomon, which is to ſhew the hours by its ſhadow in all kind of Dyals, muſt be ſet parallel to the Axis of the World: That ſo it may make

an

an Angle of Inclination with its plain ground equal to that which the Axis of the World makes with the Horizon, and that the Style is to ſtand directly North and South, and in the Meridian Line is a thing ſo commonly known, that it were needleſs to mention it.

Now if you would make an Erect South Dyal, know, That which is an Horizontal Dyal in one place, will be an Erect South-Dyal in another: As ſuppoſe you would make an Erect Dyal for the Latitude of 52: This is nothing elſe, but to make an Horizontal Dyal for the Latitude of 38 *d.* And if you make an Erect Dyal for the Latitude of 27 *d.* the ſame will be an Horizontal Dyal for the Latitude of 63 *d.* and the ſame proportion is to be obſerved in the reſt; and hence it appears that an Horizontal Dyal and a Vertical are the ſame at the Latitudes of 45 *d.* But thus much ſhall ſuffice to have ſpoken concerning drawing Hour-Lines by the Globe in an Horizontal or an Erect South Dyal.

SECT. 2.

The Analogy or Propoſitions to find them by are theſe.

PROP. 1.

For an Horizontal-Dyal.

As Radius or the ſine of 90 *d.* is to the ſine of the Latitude (ſuppoſe 51 *d.* 30 *m.*)

So

So is the Tangent of the hour, (*viz.* 15 *d.* which makes an hour in the Equinoctial.) To the Tangent of the Hour-Line from the Meridian, (*viz.* the diſtance of the hour of 1 from 12) which in this Example is 11 *d.* 51 *m.*; And ſo is 30 *d.* the Tangent of two Equinoctial hours to the Tangent of the diſtance from 12 to 2, *&c.*

PROP. 2.

For an Erect-South Dyal.

As the Radius (90 *d.*) to the Co-ſine of the Elevation (in this Example 38 *d.* 30 *m.*) So is the Tangent of any hour given, (ſuppoſe 15 *d.* for the hours 11 and 1, or 30 *d.* for 10 and 2.) to the Tangent of the hour-Line, (or to the true Diſtance of the hour-line) from the Meridian, (which in this Example is 9. 28, and 19. 46, *&c.*)

SECT. 3.

Here follows a Table taken out of *Seller*, ſhewing the diſtances of the Hour-Lines from the Meridian in ſeveral degrees of Latitude, both for an Horizontal, and alſo a South Erect Dial, as in the Latitude 54 the diſtance from 12 to 1 is 12 *d.* 41 *m.* from 1 to 2 is 25 *d.* 2 *m.* and ſo of the reſt in an Horizontal plain: But in an South Erect Dyal the Diſtance from 12 to 1 is 8.56 from 1 to 2, is 18.45, and ſo in any other Latitude you will find the degrees anſwering to each hour, which is very rea-

T dy.

dy. The Style or Cock of the Dyal must be fixed just over the Meridian Line, and must make an Angle with the Plain equal to the Height of the Pole, the hours before 6 in the morning, and after six in the Evening, may be supplyed by their opposite hours on the other side the Center.

A Table shewing the Distances of the Hour-Lines from the Meridian in these Degrees of Latitude.

Lat.	50	51.	52.	53.	54.	55.	56.	57.	58.	59.	
H.	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.	H
	2.52	2.54	2.56	2.59	3. 1	3.04	3.07	3. 8	3.10	3.12	
	5.44	5.50	5.55	6.0C	6. 4	6.09	6.13	6.18	6.22	6.28	
	8.39	8.47	8.54	9. 1	9.10	9.15	9.21	9.27	9.34	9.43	
1.	11.36	11.45	11.55	12. 5	12.13	12.22	12.32	12.39	12.49	13.00	11
	14.33	14.46	14.57	15. 8	15.19	15.31	15.42	15.52	16.02	16.16	
	17.36	17.51	18. 3	18.17	18.29	18.45	18.57	19.08	19.21	19.38	
	20.39	20.56	21.11	21.27	21.40	21.57	22.12	22.24	22.38	22.58	10
2.	23.51	24. 9	24.26	24.44	24.59	25.18	25.33	25.49	26.04	26.26	
	27. 4	27.23	27.43	28. 3	28.19	28.39	28.56	29.12	29.29	29.53	
	30.23	30.44	31. 7	31.26	31.44	32.05	32.23	32.41	32.59	33.24	
	33.51	34.12	34.34	34.57	35.15	35.39	35.59	36.16	36.34	37.00	
3.	37.27	37.50	38.12	38.36	39. 3	39.18	39.38	40.00	40.17	40.44	9
	41.10	41.32	41.57	42.19	42.39	43.03	43.23	43.42	44.02	44.30	
	44.58	45.21	45.45	46.10	46.30	46.52	47.12	47.33	47.52	48.18	
	48.54	49.18	49.41	50.05	50.25	50.48	51.08	51.27	51.45	52.12	
4.	52.58	53.22	53.44	54. 7	54.25	54.47	55.05	55.25	55.43	56.09	8

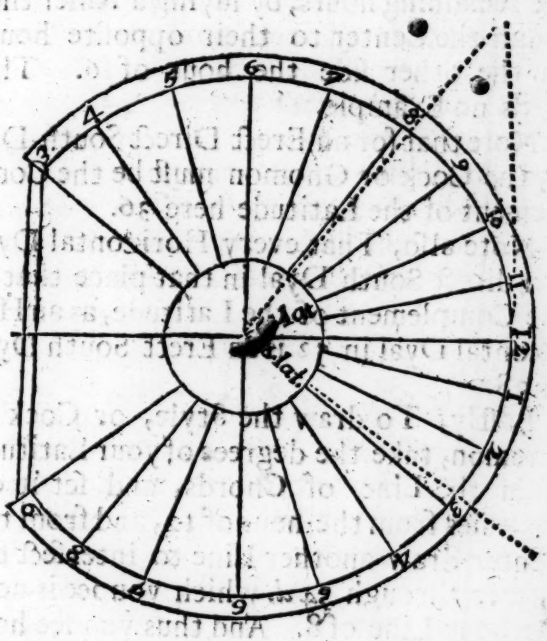
H.	50. D. M.	51. D. M.	52. D. M.	53. D. M.	54. D. M.	55. D. M.	56. D. M.	57. D. M.	58. D. M.	59. D. M.	H
	57.13	57.35	57.56	58.17	58.35	58.56	59.14	59.31	59.48	60.22	
	61.33	61.53	62.13	62.26	62.48	63.07	63.23	63.39	63.55	64.17	
	66. 5	66.23	66.40	66.57	67.11	67.28	67.42	67.56	68.10	68.29	
5	70.41	70.56	71.09	71.25	71.37	71.51	72.02	72.14	72.25	72.41	7
	75.26	75.37	75.49	76.00	76.09	76.20	76.29	76.35	76.47	76.59	
	80.14	80.23	80.30	80.38	80.44	80.52	80.58	81. 4	81.10	81.18	
	85. 2	85.06	85.10	85.14	85.18	85.22	85.26	85.31	85.34	85.39	
6	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.09	6
	40.	39.	38.	37.	36.	35.	34.	33.	32.	31.	

These Tables of Mr. *Sellars* are for an Horizontal Dyal, for Latitude 34 to 40, and from 50, to 59: The Complement whereof is for an Erect Direct South Dyal in such places of Latitude as is the Complement thereunto, as from 31 to 40, and from 50, to 56. This called by *Sellar* Tabular Dyalling easiest of any other.

South	56.	55.	54.	53.	52.	51.	50.	
Horiz.	34.	35.	36.	37.	38.	39.	40.	
	2.05	2.05	2.12	2.15	2.18	2.21	2.24	
	4.12	4.15	4.25	4.31	4.38	4.43	4.50	
	6.20	6.30	6.40	6.49	6.58	7.07	7.17	
1	8.31	8.44	8.57	9.09	9.22	9.34	9.47	11
	10.43	11.00	11.16	11.31	11.47	12.01	5.108	
	13. 2	13.21	13.41	14.19	14.35	14.55	14.00	
	15.22	15.46	16. 8	16.29	16.51	17.11	17.33	
2	17.52	18.18	18.44	19. 8	19.33	19.56	20.21	10
	20.27	20.56	21.24	21.52	22.19	22.45	23.13	
	23. 9	23.41	24.14	24.43	25.14	25.41	26.12	
	26.03	26.38	27.13	27.46	28.18	28.49	29.23	
3	29.11	29.49	30.27	31.01	31.36	32.08	32.44	9
	32.31	33.11	33.51	34.27	35.04	35.38	36.16	
	36.04	36.46	37.28	38.06	38.45	39.19	39.58	
	39.55	40.36	41.22	42.00	42.40	43.16	43,55	
4	44.03	44.48	45.29	46.10	46.48	47.24	48.04	8
	48.34	49.18	50.00	50.39	51.17	51.52	52.30	
	53.28	54.05	54.46	55.24	56.00	56.33	57.10	
	58.42	59.21	59.59	60.33	61.06	61.36	62.09	
5	64.20	64.53	65.27	65.56	66.26	66.52	67.20	7
	70.23	70.51	71.17	71.36	72.05	72 25	72,48	
	76.44	77. 3	77.22	77.38	77.38	78.05	78.25	
6	83.18	83.28	83.38	83.47	83.55	84.03	34.11	
	90.00	90.00	90.00	90.00	90.00	90.00	90.00	6

To draw an Horizontal-Dyal by this Table for 54 d. *Latitude.*

First, Take with your Compasses from a Line of Chords 60 *d.* with that distance describe a Circle, cross that Circle thorough the Center with a Diameter for a Meridian Line, and cross that at right Angles for the hour of 6, then consider for what Lati-



tude you draw the Dyal, which in this is 54, look for 54 towards the left hand of

this Table, and over againſt the Latitude, you will find 12 *d.* 14. for the diſtance of 11. and 1. from the Meridian, 25 *d.* 2 *m.* for the diſtance of the hours 10. and 2, *&c.* Theſe you are with your Compaſſes to take from your Line of Chords, and prick them upon the Dyal-Circle, and ſo draw Lines to each mark, which are the hour-lines. After you come at the hour of 6 you may draw the remaining hours, by laying a Ruler thorough the Center to their oppoſite hours on the other ſide the hour of 6. This needs no Example.

Note that for an Erect Direct South-Dyal, the Cock or Gnomon muſt be the Complement of the Latitude here 36.

Note alſo, That every Horizontal Dyal is a direct South Dyal in that place that is the Complement of the Latitude, as an Horizontal Dyal in 52 is an Erect South Dyal in 38.

Laſtly, To draw the Style, or Cock or Gnomon, take the degrees of your Latitude from the Line of Chords, and ſet it off ſidewaies from the hour of 12, and from the Center draw another Line to interſect the former thrcugh 54 *d.* which you ſee is near the hour-Line of 8. And thus you ſee how eaſie it is to draw the hour-lines by this Table for an Horizontal: The ſame may be done for an Erect South-Dyal, if you look

look for the Latitude on that ſide of the Table towards the right hand; there againſt 54 you find 8 *d.* 56 *m.* for 11. and 12. and 18.45. for 10. and 2. and ſo forwards; whereby you may underſtand to make an Erect South Dyal for any Latitude mentioned in the Table: If you pleaſe to do it by Arithmetical operation, you have alſo the Analogy, or Proportions to work them by; or if you pleaſe to do it by the Globe, you may find the hour-lines by that alſo as is ſhewed in the former pages, 407. And this is all at preſent I intend to ſay concerning finding the hour of the Day by the Sun, or making Sun-*Dyals*: I ſhall only ſhew you how to find the hour of the Night by the Moon and Stars, and ſo bid good Night, and conclude this Mathematical Manual.

SECT. 4.

Firſt then, By the Moon.

You muſt underſtand, that by knowing the Prime you may find the Epact, and by knowing the Epact you may find the Age of the Moon; by knowing the Age of the Moon, you may find the time of her coming to the South; and by knowing the time of her coming to the South, you may find the hour of the Night by the ſhadow of the Moon upon a Sun-*Dyal*. Therefore of theſe in order.

PROP. 1.

To find the Prime or Golden Number, Divide the year of our Lord by 19, and to that which remaineth after the diviſion add 1, the Product is the Prime Number for all that year, beginning the 1*ſt. January*: As 1673 ⌊ 88 $\frac{1}{19}$, ſo $\frac{1}{1}$ 2 is the Prime.

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PROP. 2.

By the Prime, To find the Epact.

Multiply the Prime of the year by 11, and Divide the Product by 30, and the Remainder or Fraction is the Epact. If it be juſt 30, then is the Prime and the Epact both one; if ſhort of 30, that Number whatever it is, is the Epact. Note that the Prime changes 1. *January*, and the Epact 1. *March*.

PROP. 3.

By the Epact, To know the Age of the Moon.

Add to the day of the Month the Epact, and ſo many dayes more as are Moneths from *March* to the Moneth you are in, including both Months; and if they come not to 30, ſo much the Moons Age: If they paſs 30, Take away 30, and the Remainder is the Moons Age. This is when the Month hath 31 dayes, but if it have only 30 dayes, deduct 29 *&c.* from the exceſs above 29, by the ſame reaſon.

PROP.

PROP. 4.

By the Age of the Moon, to find the time of her coming to the South.

Multiply the Moons Age by 12, and divide the Product by 15, the Quotient will ſhew the hour of the Moons coming to the South; and if any thing remain, multiply that by 4, and it will ſhew the Minutes to be added to the hours of the Quotient, and ſo you ſhall have the time of the Moons coming to the South: Or by the Table following you may readily find the time of the Moons coming to the South.

Dayes

The Dayes of the Moons Age.

Days	H.M.
1	0.45
2	1.38
3	2.26
4	3.15
5	4. 3
6	4.53
7	5.41
8	6.30
9	7.15
10	8. 8
11	8.56
12	9.45
13	10.34
14	11.23
15	12.11
16	Afternoon.
17	1.10
18	1.49
19	2.38
20	3.26
21	4.15
22	5. 3
23	5.53
24	6.41
25	7.30
26	8.19
27	9. 8
28	9.59
29	10.45
30	11.34
	Morning.

The Hours of the Moons Age.

Hours	H.M.
1	2
2	4
3	6
4	8
5	10
6	12
7	14
8	16
9	18
10	20
11	22
12	24
13	26
14	28
15	30
16	32
17	34
18	36
19	38
20	40
21	43
22	45
23	47
24	49

The use of the Table according to the Age of the Moon, Take the Hours and Minutes oppofite thereunto in the Table; adding for every hour 2 Minutes more for her mean motion; and the Total ſhall ſhew the hour of the Moons coming to the South.

For Example.

Suppoſe the Moons Age be any day at Noon 10 days and 8 hours old. Look in the Table, and againſt 10 dayes you find ——————— 8 *h.* 8 *m.*
for the 8 odd hours — 16 *m.*
To which 8 Hours 24 *m.* allowing 2 Minutes for each Hour for her mean motion is — 17 *m.*
All which added together ſhews ſhe comes to the South at ——— 8 *h.* 41 *m.*

To

To know at what hour the Moon comes to the South, by the following Table.

Dayes after the Change or Full the Moon is South at.

0.	1	2	3	4	5.	6.	7.
12	12.48	1.36	2.24	3.12	4.0	4.48	5.36

Dayes after either of the Quarters the Moon is South at

16	6.48	7.36	8.24	9.12	10.0	10.48	11.36

To know the Hour of the Night by the Moons ſhadow upon a Sun-Dyal.

D	H.	M.	☾
1	0	0	29
2	10	24	28
3	9	36	27
4	8	48	26
5	8	0	25
6	7	12	24
7	6	24	23
8	5	36	22
9	4	48	21
10	4	0	20
11	3	12	19
12	2	24	18
13	1	36	17
14	0	48	16
15	0	0	15

First know the Age of the *Moon*, and if it increaſe, from the ſhadow ſubſtract theſe hours and Minutes: If it decreaſe, add the ſame to the ſhadow, and it will give you the true hour.

PROP.

PROP. 5.

By knowing the time of the Moons coming to the South, To know the time of the Night by the ſhadow of the Moon upon a Sun-Dyal.

Obſerve by a Sun-Dyal as if you would ſee what a Clock it were by the Moon, and obſerve how much the ſhadow of the Moon doth either want, or is paſt the hour of 12 upon the Dyal: For ſo much it doth want of, or is paſt the time of the Moons coming to the South. *As for Example*,

Suppoſe as before the Moon did come to the South at 8 hours, 41 minutes after noon or at night, and the ſhadow of the Moon upon the Dyal were at 10. This wants 2 hours of 12, and therefore it wants 2*h.* of 8*h.* 41*m.* ſo that it is ſix of the Clock, and 41 *m.* But if the ſhadow had been at 2 upon the Dyal, then you muſt have added 2 *h.* to the Moons coming to the South, and ſo it had been 10 *h.* 41 *m.* at night: The like is to be done by any other hour whatſoever.

SECT. 5.

To know the hour of the Night by the Stars.

1. To do this by the Globe, Rectifie the Globe to the Latitude of the place, and bring the Suns place to the Brazen Meridian, and the Index of the hour Circle to 12: Then bring any ſtar to the ſame Altitude on the Globe that it hath in Heaven, and the

the Index of the hour-Circle will point to the hour of the Night.

2. *By Mr.* Gunters *Nocturnal.*

This Nocturnal is a very eaſie and ready way to find the hour of the Night by the ſtars wherein the Center upon which it moves is ſuppoſed the North-Pole, having ſeveral conſtellations near the ſame Pole deſcribed upon the Nocturnal, with a Circle of Moneths fixed lying without the Rundle, as alſo an Hour-Circle of twice 12 *h.* one of the 12 ſignifying 12 at Noon, the other 12 at Midnight; and the line ſuppoſed to be drawn betwixt 12 and 12, thorough the Center is the Meridian: Now for the uſe thereof, firſt look up to Heaven to the North Pole, and obſerve what ſtars are upon or near the Meridian in Heaven: Then place the ſame ſtar to the like ſcituation upon the Rundle, and againſt the day of the Moneth you will find the hour of the Night.

3. *In the third place, To find the hour of the Night by the Stars coming to the Meridian; their Riſing, and Setting, and Right Aſcenſion.*

How to find the Right Aſcenſion of any ſtar by the Globe is taught, *pag.* 363, and is no more but inſtead of bringing the Suns place to the Meridian, you muſt bring the ſtar, (whoſe Right Aſcenſion you ſeek) to the

the Meridian, and the number of degrees contained betwixt the vernal colure and that degree of the Æquinoctial which comes to the Meridian with the ſtar, is that ſtars right Aſcenſion: Alſo by the Globe to find the time of any ſtars Riſing or Setting, or coming to the Meridian, is taught *pag.* 364. 364, How to find the Right Aſcenſion of any ſtar by Theakers Planiſphear is ſhown *pag.* 374: As alſo their riſing and ſetting, and their coming to the Meridian, is ſhown *pag,* 375.

To find the Suns Right Aſcenſion is the ſame both by the Globe and Planiſphear, by bringing the Suns place to the Meridian in ſtead of bringing the Star to the brazen Meridian on the Globe, or the paper Meridian of the Planiſphear, as is explained in the forementioned places, *pag.* 362, 374. you have alſo the Analogy or Proportion to work it by Arithmetically, *pag.* 350. So need not here repeat any of them.

But ſhall ſhew you how by the Right Aſcenſion of the Sun and ſtars to find Arithmetically the time of the ſtars coming to the South, and thereby the ſtars hour, and by all theſe the hour of the night.

1. *To find at what hour any Star will be upon the Meridian.*

Add the Complement of the Suns Right Aſcenſion for the day propoſed to the Right Aſcenſion of the Star, the ſum of them together will be the time of that ſtars being upon the Meridian.

For

For Example. h. m.

The Bulls Eye 1ſt. *January* Right Aſcenſion.——4 17
Compl. of the Suns Right Aſcenſion is——4 25

The Bulls Eye upon the Meridian.——8 42
That is, 42 Minutes paſt 8 at Night.

It is noted by *Norwood* that any ſtar in 14 dayes comes to the Meridian juſt an hour later, and in a Moneth two hours.

2. *To find the Riſing and Setting of the Stars.*

The Semidiurnal Arch of any ſtar, which is half the time the ſtar appears above the Horizon, ſubſtracted from the time of the ſtars being upon the Meridian, giveth the time of the ſtars Riſing, and the ſame being added thereunto, giveth the time of the ſtars ſetting; and knowing the time of the ſtars riſing or ſetting, and coming to the Meridian; and ſeeing any of the ſaid ſtars riſe, ſet, or come to the Meridian in Heaven, hereby you know the Exact hour of the Night.

3. *But to find the hour of the Night at any other time, you muſt know the ſtars hour.*

Now the ſtars hour is the horary diſtance, or the number of hours that the ſaid ſtar is diſtant from the Meridian, or ſo many hours as the ſtar is ſhort of, or paſt the Meridian.

Now

Now to find the Hour of the Night at any time, by any of the Stars.

Add the Stars Hour, the Stars Right Aſcenſion, and the Complement of the Suns Right Aſcenſion; all three together, the Sum is the hour of the Night. As for Example.

The Bulls Eye 11*th* of *December.*

The Stars Hour is	8	56
The Stars Right Aſcenſion is	4	17
The Complement of the Suns Right Aſcenſion.	6	9
Somme	19	22

Now becauſe this exceeds 12, you muſt deduct 12 from it; ſo that the true hour of the Night is 7 *h.* 22 Minutes: And note further, That when this Sum is ſhort of 12, it ſhews the hour at Night; when it exceeds 12, it ſhews the hours next Morning: So that in this Example it is 7 *h.* 22 *m.* in the Morning; and herewith I ſhall conclude.

Having finiſhed what I intend to ſay concerning *Aſtronomy*, and the Reſolution of theſe Aſtronomical Queſtions of the Sphear, being the third and laſt Mathematical Liberal Science according to my Diviſion of them in the beginning: I ſhall herefore here ſet up a *Non Ultra* at this time. Having (I hope) in ſome meaſure ſhewed you, How to Meaſure the *Earth*, the *Sea*, and the *Heavens*, principally and eſpecially by *Trigonometry*. Therefore in alluſion to a *Triangle*, the moſt excellent of Geometrical Figures, and the other *Ternaries* mentioned here, and elſewhere, *pag.* 2, 3. I ſay in alluſion to theſe. To the Tri-vn Deity. The Trinity in Unity, and Unity in Trinity, who created All theſe Things of Nothing in Number, Weight, and Meaſure, by his Infinite Wiſdom and Power, and doth Uphold and Govern All by his Providence, and deſign'd all for his Glory, to him be all the Glory and Honour, and Adoration *in Secula Seculorum.* AMEN.

FINIS.